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Two Compact Smith Charts: The 3D Smith Chart and a Hyperbolic Disc Model of the Generalized Infinite Smith Chart

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Abstract. The paper is describing and presenting the recent advances in the 3D Smith chart representations and applications and then it proposes a new conceptual model for the extended 2D Smith chart based on hyperbolic geometry by mapping the generalized Smith chart in the unit disc using the stereographic projection from a hyperboloid (Poincare disc model).

1. Introduction

The Smith chart, also called Reflection Chart, Circle Diagram, Immitance Chart and Z plane chart in its initial stages, was first published in 1939 [1] and refined throughout the years [2] before becoming a universal tool in microwave engineering design and measurement stages. So entrenched is the Smith chart in microwave engineers' conceptualization that despite their powerful and highly sophisticated computational capability, both the modern computer-aided design software and the computer controlled microwave measurement equipment continue to present results on Smith chart overlays [3]. To have a finite and practical size, the classical 2D Smith Chart is constrained to the unit circle. Hence, loads with reflection coefficient magnitude greater than 1 cannot be plotted. These loads often appear in active circuits and in lossy transmission lines with complex characteristic impedances [4]. The reason for seeking an expansion was determined by the desire to have a unique chart suitable for "including also the negative impedances" (which have the modulus of the reflection coefficient bigger than one) without sacrificing the usual benefits the Smith chart usually offers. Recent attempts to overcome this limitation failed to provide a simple unitary model for it, with neat solutions being based on empirical intuition [5] or with very interesting solutions based on difficult manipulations but which lose many of the planar Smith Chart properties [6, 7].

In 2011 the authors introduced the 3D Smith chart model [8], refined and extended in [9-12] in order to represent all possible reflection coefficients on the unit ball surface (see Fig.1).

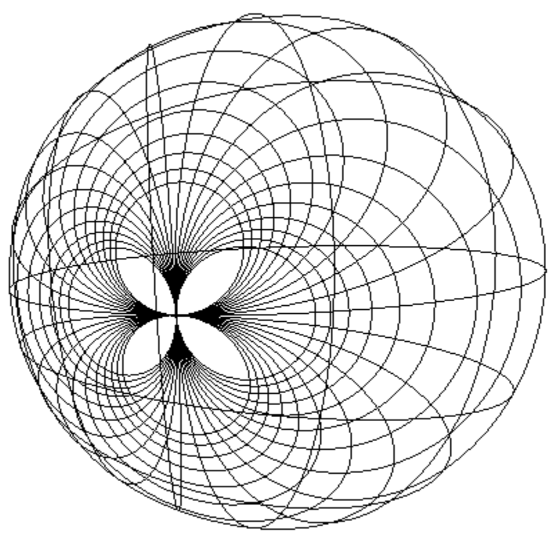
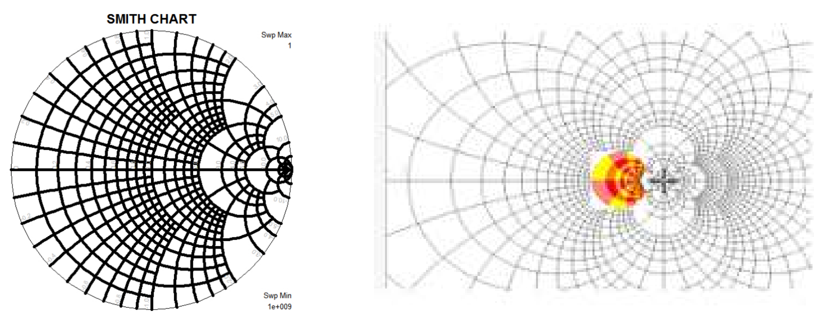


Fig. 1. 3D Smith chart representation (`www.3dsmithchart.com`).

2. 3D Smith chart basic theory

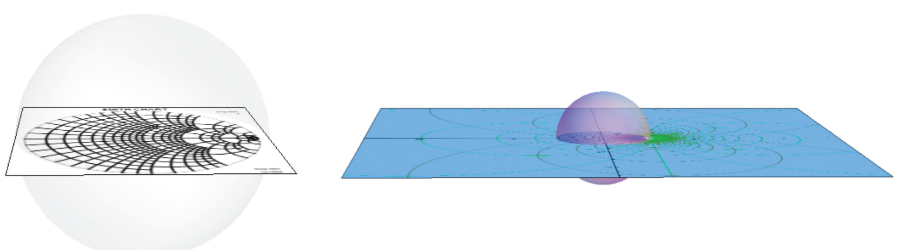
2.1. 3D Smith chart construction

The extended reflection coefficient plane (Fig. 2b) is mapped stereographically through the South Pole [8-9] on the surface of a unit sphere (Fig. 3). As a result, the classical 2D Smith chart (considered in the equatorial plane unit circle) including the passive loads is mapped stereographically into the North hemisphere, while the circuits with negative resistance (that are outside the classical planar Smith chart) are mapped into the South one. The East hemisphere is the place of inductive circuits, whereas the West hemisphere hosts the capacitive circuits. Meantime, the Greenwich meridian is the locus of pure restive circuits.



a) b)

Fig. 2. (a) Smith chart representation (limited to the unit circle where the magnitude of the voltage reflection coefficient is smaller than unity). (b) Extended Smith chart including all types of circuits extends to infinity in 2D.



a) b)

Fig. 3. (a) Smith chart is considered in the interior of the equatorial plane and is mapped stereographically through the South pole in the North hemisphere; (b) Extended Smith chart: circuits with magnitude of the reflection coefficient bigger than one are in the equatorial plane outside of the unit circle and mapped in the South hemisphere.

2.2. 3D Smith chart tool

The current version of the program (including all its components - inputs, mathematical calculations, graphical user interface and 3D visualizations) is implemented in Java and thus can be run on most platforms and operating systems. Future developments will include:

– extensions regarding the types of visualization and analysis;

– supporting more powerful, natural interactions with the 3D chart, using hand-tracking devices;

– implementations as plug-ins compatible with all major software tools currently used in the practice of circuits design and analysis.

3. 3D Smith chart applications

3.1. 3D Smith chart power wave and voltage refection coefficients

As described in [10] the magnitude of the voltage reflection coefficient [13–14] can exceed unity in the case of complex impedances while its value is different from the power wave reflection coefficient [15] once complex port impedances are used (see Fig. 4).

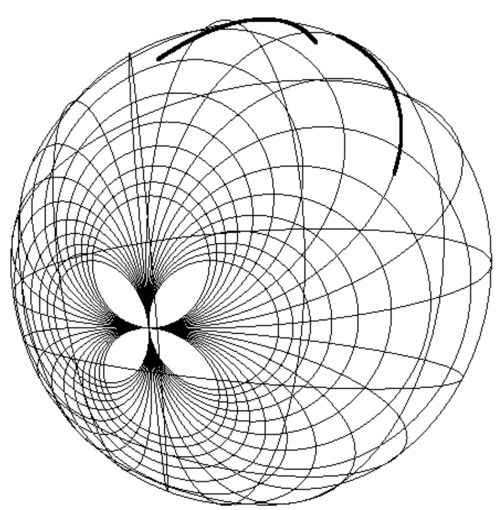


Fig. 4. The power wave and voltage reflection coefficient are computed on the 3D Smith chart in the case of complex input and output ports.

3.2. 3D Smith chart stability circles

In the *reflection*-plane, and therefore in the 2D Smith chart, the boundary between stability and instability regions are circles. The center and radius of the source and load stability circles can be easily computed from the S-parameters of the transistor (or, in general, the active circuit providing the amplification. In most cases, however, one or both circles include active loads, thus being partially outside of the 2D Smith chart. In a wide frequency range, this problem generates visualization problems in 2D due to the scaling required to be able to plot the stability circles, identify the problematic regions and look for a possible solution. The 3D Smith chart does not require any type of scaling, since all the active and passive loads are successfully represented in a bounded surface. Moreover, and due to properties of the stereographic projection, the stability circles in the planar Smith chart transforms into circles in the Riemann Sphere [9]. An example of various stability circles can be seen in Fig. 5.

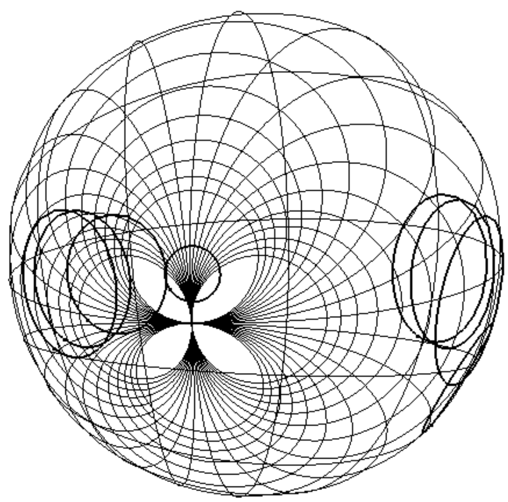


Fig. 5. Stability circles on the Smith chart and 3D Smith chart (if they go out of the Smith chart circumference they are mapped south).

3.3. 3D Smith chart unilateral constant transducer power gain circles

In the design of radio frequency (RF) amplifiers and active modulators (*e.g.* frequency multipliers, mixer sand small-shift frequency translators), the unilateral constant power gain circles represent the loci of source and load impedances on a conventional2-dimensional (2D) Smith chart. Displaying power levels, or observing the Smith chart coverage for a specific power level, leads to poor visualization when using 2D plots; gain and mismatch circles always converge to point circles (*i.e.*, circles of zero radius) on the 2D Smith chart In [11] has been shown that unilateral constant power gain circles (source circles in our example) are a subfamily of the Apollonius circles, with respect to $S11^*$ and $1/S11$. Relative power levels on the 3D Smith chart have been plotted for the first time, as a newly proposed visualization tool (see Fig. 6), overcoming traditional limits associated with contour plots on the traditional 2D Smith chart.

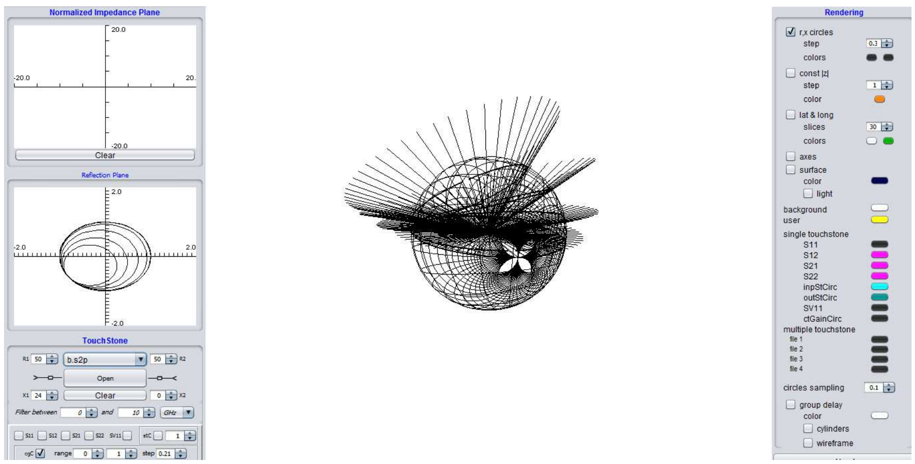


Fig. 6. Transducer power gain circles on the 3D Smith chart.

4. Hyperbolic Smith chart

In this paragraph we introduce a new generalized Smith chart [16] by mapping the entire reflection coefficients plane in the unit disc (of a hyperbolic reflection coefficients plane) and thus avoiding the usage of 3D embedded surfaces for the compactification of the generalized Smith chart. In this purpose we use:

1. an immersion (that can be loosely described as a vertical pull) from the 2D generalized Smith chart to the positive part of two sheet hyperboloid (see Fig. 7);

2. a stereographic projection [17] from the two sheet hyperboloid onto the unit disc (used in the Poincare unit disc model of hyperbolic geometry, here for a compactification of the infinite regions) (Fig. 8).

The hyperbolic Smith chart embodies all possible circuits in the unit disc, the unit circle is represented by circuits with infinite magnitude of the voltage reflection coefficient while the interior of the unit circle is represented by circuits with a finite magnitude of the reflection coefficient. The classical Smith chart lies in the interior of the 0.414 radius circle of the complex plane.

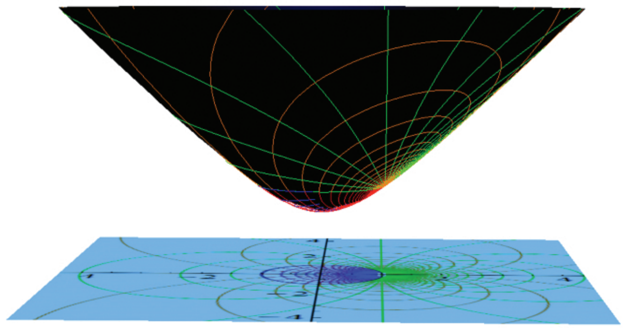


Fig. 7. Preliminary construction: Generalized Smith chart mapped on the two sheet hyperboloid (it extends to infinity but on the upper sheet of the hyperboloid) [16].

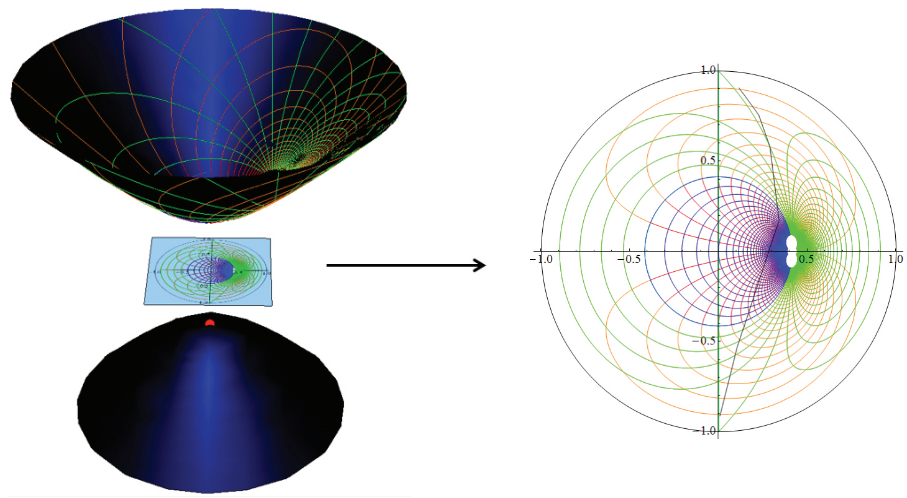


Fig. 8. Hyperbolic Smith chart: The upper sheet of the hyperboloid in Fig. 7 is projected stereographically from point (0,0.-1) onto the unit circle [16] using Poincare's model of hyperbolic geometry.

5. Conclusion

The paper first revisited the recent applications of the 3D Smith chart model [8]: in amplifier stability analysis, complex port matching, unilateral transducer constant power gain circles representations showing illustrative qualitative examples. In the end it introduces a hyperbolic generalized Smith chart model [16]. The classical Smith chart is mapped in the interior of the 0.414 circle of the hyperbolic complex plane (circuits with voltage coefficients magnitude lower than unity are mapped inside of this region while circuits with magnitude M: (1<M<Infinity) are mapped in the region between the 0.414 radius circle

and unit circle. Nevertheless inductive circuits are mapped above the real axes and capacitive below it. The paper thus presented two compact models of the generalized (infinite extending) Smith chart: the 3D (spherical one which keeps the circular form of the classical Smith chart and the hyperbolic one which lies inside of the unit disc but which distorts the inductances and capacitances contours forms).

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