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On the light pressure induced in a medium by the non-uniform light intensity distribution

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Abstract
We describe the behaviour of the light pressure due to the gradient force that appears in the case when a non-uniform light beam propagates inside a medium. We show its behaviour both inside a medium and at the interface between two media, as well as consider other sources that contribute to this light pressure. We show also that this pressure may contribute to the heating of the propagation medium when the beam is in the microwave or far-infrared region. Discussions regarding some possible applications, as well as a comparison with the usual light pressure, are presented.

Keywords: light pressure, microstructures, microsystems, nanostructures

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The first experimental evidence of light pressure was discovered by Lebedev in 1901 [1]. Today we can observe great interest in using light pressure for moving objects at a micro-and nanoscale, as well as in applying it for light detection [2–4]. Even if there are described so many applications, there are still debates on the correct interpretation and expression of light pressure, starting from the electromagnetic field parameters. Two such papers are [5, 6]. There are two forces that appear as a consequence of the Lorentz force. The first one, given by the Lorentz force acting on charges, is proportional to the $\boldsymbol{E} \times \boldsymbol{H}$ product of the electromagnetic wave. The second one, acting on the dipoles, is given by the gradient of the squared electric field. Details can be found in [7]. The former kind of light pressure is usually associated with the photon momentum, when the corpuscular picture is used. It appears mainly at the interface between two media and, in some cases [5], inside an optical medium. Throughout this work I shall call this type of pressure the conventional light pressure. The latter kind of pressure appears both at interfaces and inside the optical medium only when we have a non-uniform intensity light distribution, as is proven further. I shall consider only this type of pressure, known in the literature as the gradient force.

In this paper I present the theoretical description of the behaviour of this kind of light pressure, which has components transverse to the light propagation direction and which is due to the non-uniform intensity distribution of the light beam across its section. The paper deals with the detailed treatment of this gradient force, regarding its behaviour at interfaces, in thin films as well as considering other sources that may contribute to it.

I mention that the force exerted on individual atoms and molecules by non-uniform light intensity distributions was intensively studied previously, with regard to laser micromanipulation and atom trapping. With respect to these studies, we present the interaction of light with macroscopic bodies (i.e. those whose dimensions are greater than the wavelength and the beam size is less than the body size) and deduce the macroscopic behaviour of light pressure at this scale. To our knowledge, the results obtained by us are novel and have not been reported previously in the literature. As will be seen further, the behaviour of this type of light pressure, at the macroscopic scale is different from that of conventional, photon-momentum-based, light pressure [5].

2. Theoretical background

2.1. Generalities

We consider a perfectly transparent, homogeneous and nonmagnetic medium having a refractive index equal to $n(\lambda)$, where λ is the wavelength of light. Through this medium a light beam having an intensity distribution $I(r, \varphi)$ propagates, where r is the distance from the beam centre to the current point and φ is the polar angle. We look at the beam inside this medium and, for reasons of simplicity, do not take light pressure exerted by the beam when entering in the medium. The light propagates along the x direction. We consider that the beam is sufficiently long so as to occupy all the distance from the entrance to the exit facet of the medium. The medium is extended spatially (lateral) much more than the beam size.

The model we consider in this paper does not apply to very dilute systems. By very dilute systems we understand systems whose molecules have a mean free path of at least a tenth of the incident light wavelength. Such a situation may appear in the case of rarefied gases. The model can be extended also to this situation by considering electromagnetic radiation with a longer wavelength, such that the mentioned condition be fulfilled.

We may start from the work of [7] or from the equation describing the volume density of force [8] acting on a dielectric:

$$\vec{f} = -\operatorname{grad}[P_0(\rho, T)] - \tfrac{1}{2}\vec{E}^2 \operatorname{grad}(\varepsilon) - \tfrac{1}{2}\vec{B}^2 \operatorname{grad}(\mu) + \operatorname{grad}\left[\frac{1}{2}\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \vec{E}^2 + \frac{1}{2}\rho\left(\frac{\partial\mu}{\partial\rho}\right)_T \vec{B}^2\right] + \frac{\varepsilon\mu - 1}{2c}\frac{\partial}{\partial t}\left(\vec{E}\times\vec{B}\right), \quad (1)$$

where $P_0(\rho, T)$ is the external pressure applied to the medium (ρ is density and T is temperature), ε is the electric permittivity and μ is the magnetic permeability, while **E** and **B** stand for the electric field and magnetic field, respectively.

The first term on the right-hand side of the equation stands for the volume density of the force produced by external pressures (mechanical load). In our case, this term is equal to zero, since we have not considered any action of the external loads. The following two terms are due to the inhomogeneous nature of the medium as regards the electrical and magnetic properties, respectively. Since we consider a homogeneous medium, these two terms also vanish. The last term in the above equation is the Abraham force and is responsible for the usual, photon-momentum-based, light pressure. It is extensively studied in the literature (see [5] and references therein). We will not consider it in this paper, since its behaviour is well documented. The last two terms from the right-hand side of the equation can also be obtained from the density of force presented in [9].

The only term that we present in this paper is the fourth term, for a non-magnetic medium (i.e. $\mu = \mu_0$). In this case, equation (1) reduces to

$$\vec{f} = \operatorname{grad}\left[\frac{1}{2}\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T \vec{E}^2\right] = \frac{1}{2}(\varepsilon - \varepsilon_0)\operatorname{grad}\left[\vec{E}^2\right]. \quad (2)$$

Here ε_0 is the permittivity of vacuum. However, for most materials $\mu = \mu_0$ at optical frequencies [10], so that the magnetic component of this kind of light pressure vanishes. In equation (2) we have supposed that the polarization **P** of the medium depends linearly on the density ρ. This is true for gases and most liquids, as well as for some solids [9].

We point out that working with the volume density of force (derived from the gradient of the total energy of the medium) is equivalent to working with the Maxwell stress tensor, as shown by [11].

We consider the light field as having the form:

$$E(r, \varphi, x) = E_0(r, \varphi)\sin(\omega t + kx), \quad (3)$$

where ω is the angular frequency of the light beam and k is its wavevector. We obtain:

$$E^2(r, \varphi, x) = \frac{1}{\varepsilon_0 cn} I(r, \varphi)[1 + \cos(2\omega t + 2kx)], \quad (4)$$

where c is the light speed in vacuum and n is the refractive index. From (1) and (4) we obtain:

$$f(r, \varphi) = \frac{n^2(\lambda) - 1}{2nc}[\operatorname{grad}(I(r, \varphi)) + \operatorname{grad}[I(r, \varphi)\cos(2\omega t + 2kx)]] = f_0 + f_{\mathrm{opt}}, \quad (5)$$

where we have considered that, at optical frequencies, $\varepsilon_r(\lambda) = n^2(\lambda)$. In the above expressions, f_0 stands for the low-frequency component of the force density while f_{opt} represents the contribution oscillating at optical frequencies.

f_0 is given by the expression

$$f_0(r, \varphi) = \frac{n^2(\lambda) - 1}{2nc}\operatorname{grad}[I(r, \varphi)]. \quad (6)$$

In a certain sense, f_0 represents the mechanical equivalent of the optical rectification.

The low-frequency f_0 is of interest for most practical applications and generates the light pressure due to the non-uniform distribution of light intensity across its section. If the intensity is modulated in time, the same kind of modulation appears for f_0.

It is observed from equation (6) that light pressure appears along all those directions where an intensity gradient exists.

2.2. *Light pressure inside a medium*

In this case there is no spatial variation of the electrical permittivity, so that equation (6) of the density of force may be used as it is. The medium being transparent, we have no gradient of intensity along the propagation direction.

In the case of a beam with circular symmetry, the pressure difference $p_{\mathrm{i}}(q, r)$ between points located at a distance 'q' and 'r', respectively, from the beam centre, is obtained by direct integration of $f(r, \varphi)$ along the radius 'r' and is given by

$$p_{\mathrm{i}}(q, r) = \frac{n^2(\lambda) - 1}{2n(\lambda)c}\int_q^r [\operatorname{grad}_{r'}(I(r'))]\,\mathrm{d}r' = \frac{n^2(\lambda) - 1}{2n(\lambda)c}[I(r) - I(q)]. \quad (7)$$

From expression (7) it is observed that light pressure depends on the intensity difference between the two points of interest. For the case when a beam of uniform intensity illuminates the medium, light pressure vanishes. This is a specific feature of this type of light pressure.

If we take 'q' at the margin of the beam where $I(q) = 0$, we obtain for $p_{\mathrm{i}}(r)$ a value equal to

$$p_{\mathrm{i}}(r) = \frac{n^2(\lambda) - 1}{2n(\lambda)c} I(r). \quad (8)$$

From equation (7) it is observed that light pressure is directed towards the region of higher light intensity.

If we consider a refractive index $n = 2$, then we obtain for the light pressure $p_i(r)$ a value of $2.5 \times 10^{-9} \times I(r)$ N m^{-2}. This is a value of the same order of magnitude as that of the conventional light pressure. According to this consequence, the total light pressure is given by the sum of two components:

$$p = p_u + p_i \tag{9}$$

where p_u is the conventional light pressure and p_i is the light pressure due to the non-uniform intensity distribution of the light beam across its section.

From equation (8) we note two important aspects:

- this type of pressure, for a transparent medium, is always perpendicular to the propagation direction. There is no light pressure component directed along the propagation direction, except for the case of absorbing media and thin films.
- if $I(q) = I(r)$ for any 'q' in equation (7), then there is no pressure in that region. In other words, a uniform intensity beam exerts no light pressure of the type described in this paper.

Because the value of p_i is very small, we consider as a good approximation the fact that the pressure of light does not deform, respectively stress, the medium, so that the electrical permittivity ε of it remains constant. In other words, nonlinear effects due to the variation of optical properties of the medium as a consequence of the light pressure can be neglected.

2.3. Light pressure of a light pulse

We present briefly in this section the pressure that appears in the case of short light pulses, whose spatial extension is less than the distance between the two facets of the medium. In this case, even in perfectly transparent medium, a pressure appears along the propagation direction (oriented toward the higher intensity region of the pulse), in the region where the beam is located at a certain instant of time, due to the non-uniform distribution of light intensity inside the pulse (from front margin to rear margin). In this case, the spatial region containing, at a certain moment of time, the light pulse is compressed hydrostatically (i.e. from all directions).

As the pulse propagates inside the medium, a mechanical wave is generated because of the compression–relaxation cycle (only one cycle for each point) that appears in a given point when the light pulse passes through it. This pulse travels at the speed of light inside the medium, a speed that is much higher than the sound speed inside the same medium. Because of that we may expect that a mechanical wave, similar to that produced by supersonic jets or in the case of the Čerenkov effect, be produced inside the medium when such a light pulse propagates through it. The amplitude of the mechanical wave is given by the pressure expressed in (8) and, as numerical estimates from above show, is of small value.

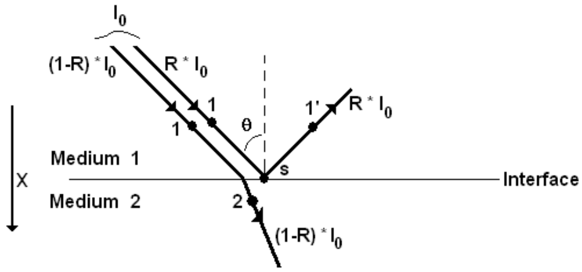


Figure 1. Light incident on the interface between two media.

2.4. Light pressure at the interface between two distinct media

Now we will consider the case when light passes from one optical medium to the other and see how light pressure due to gradient force appears as a consequence of crossing the interface.

Because in general both E and ε vary at the interface, we will rewrite (1) in the form:

$$f = \tfrac{1}{2}\,\mathrm{grad}\left[(\varepsilon - \varepsilon_0)\,E^2\right] - \tfrac{1}{2}E^2\,\mathrm{grad}\,(\varepsilon)\,. \tag{10}$$

This form of (1) ensures an easier integration when crossing the interface.

Before presenting the case of s-polarized and p-polarized beams, respectively, we present the effect of light reflection on the light pressure. We point out that we refer only to light pressure arising from beam intensity or medium inhomogeneities (in the case of interfaces).

For this purpose, we consider that an incident beam of intensity I_0 is incident onto the surface of separation between two media having a permittivity ε_1, respectively ε_2. These media are as above, i.e. are homogeneous, isotropic, linear and transparent. Let R be the reflectivity coefficient at the interface. We decompose formally the incident beam into two sub-beams, one of intensity RI_0 and the other of intensity $(1 - R)I_0$. The reflected beam has an intensity equal to RI_0. The situation is depicted in figure 1.

In figure 1, θ is the angle of incidence. Let F be a primitive of f, that is:

$$F = \int f\,\mathrm{d}x. \tag{11}$$

We remember that all the gradients are normal to the interface, that is, are directed along the X axis. Let us consider the contribution of reflectivity to the light pressure p_R. For this purpose, we will integrate (10) along the $1 \to s \to 1'$ path (depicted in figure 1). We consider the points 1 and $1'$ as being very close to the interface, while point s lies directly on the interface. The interface may be considered as a very thin layer whose thickness tends to zero. We obtain

$$p_R = \int_1^{1'} f\,\mathrm{d}x = \int_1^{s} f\,\mathrm{d}x + \int_s^{1'} f\,\mathrm{d}x = F(s) - F(1) + F(1') - F(s) = 0. \tag{12}$$

This means that the reflected beam does not contribute at all to the light pressure. It is an important feature of this type of light pressure. The only part of the beam that contributes

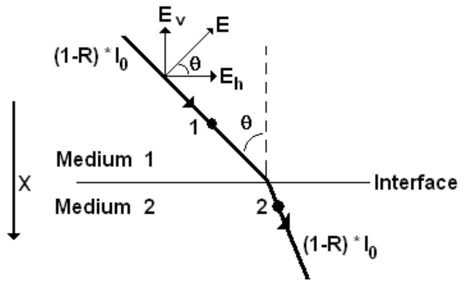


Figure 2. Geometry of the p-type beam.

to the light pressure at the interface is the transmitted beam of intensity $(1-R)I_0$. This can also be understood starting from the fact that the density of force given by equation (1) derives from a gradient. The gradient from $1 \rightarrow s$ has an opposite sign (and equal magnitude) with the gradient from $s \rightarrow 1'$.

Now let us consider the light pressure of the transmitted beam for the two types of beam polarizations, s-type and, respectively, p-type.

2.4.1. s-type beam. In this case, the beam is polarized parallel to the interface. The electric field is continuous at the interface and thus we have $E_1 = E_2 = E$, where E is corrected for the reflection. We integrate (10) from point 1 in the first medium to point 2 in the second medium, these two points being very close to the interface (the interface considered as a virtual layer whose thickness tends to zero). The pressure p_s in this case is

$$p_s = \int_1^2 \tfrac{1}{2}\,\mathrm{grad}[(\varepsilon - \varepsilon_0)E^2]\,\mathrm{d}x - \int_1^2 \tfrac{1}{2}E^2\,\mathrm{grad}(\varepsilon)\,\mathrm{d}x. \quad (13)$$

Taking into account the continuity equation for E and making the computation, we obtain that

$$p_s = 0. \quad (14)$$

This is another distinctive feature of this type of light pressure, namely that an s-polarized beam does not exert pressure at the interface between two different media.

2.4.2. p-type beam. Let us consider the beam polarization as in figure 2.

We have:

$$\begin{aligned} E_h &= E\cos(\theta) \\ E_v &= E\sin(\theta)\,. \end{aligned} \quad (15)$$

For E_h it is like the s-type beam. Thus the pressure due to this component is equal to zero. Thus, there remains only the E_v component. But this component varies when crossing the interface, thus we use the relation:

$$D_v = \varepsilon E_v \Leftrightarrow E_v = \frac{D_v}{\varepsilon} \quad (16)$$

since D_v, the normal component of D, is continuous at the interface. Thus, (10) becomes:

$$p_v = \int_1^2 \frac{1}{2}\,\mathrm{grad}\left[(\varepsilon - \varepsilon_0)\frac{D_v^2}{\varepsilon^2}\right]\mathrm{d}x - \int_1^2 \frac{1}{2}D_v^2\frac{1}{\varepsilon^2}\,\mathrm{grad}\,(\varepsilon)\,\mathrm{d}x. \quad (17)$$

By direct computation, we obtain

$$p_v = -\frac{1}{2}\varepsilon_1 E^2\sin^2(\theta)\frac{n^2-1}{n^2}\left(2 - \frac{\varepsilon_0}{\varepsilon_1}\frac{n^2+1}{n^2}\right), \quad (18)$$

where $n = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ is the refractive index of medium 2 relative to medium 1. If medium 1 is a vacuum, then the pressure exerted by light is:

$$\begin{aligned} p_v &= -\frac{1}{2}\varepsilon_0 E^2\left(\frac{n^2-1}{n^2}\right)^2\sin^2(\theta) \\ &= -\frac{1}{2nc}\left(1-R_p\right)\left(\frac{n^2-1}{n^2}\right)^2 I\sin^2(\theta)\,, \end{aligned} \quad (19)$$

where R_p is the reflection coefficient (intensity) for the p-polarized beam and I is the incident light intensity.

R_p is given by [14] the expression:

$$R_p = \frac{\tan^2(\theta - \theta')}{\tan^2(\theta + \theta'),} \quad (20)$$

with θ' the angle of refraction given by the expression

$$\sin\theta' = \frac{1}{n}\sin\theta. \quad (21)$$

From (18) and (19) it is observed that normal light pressure p_v vanishes when the incidence angle is equal to zero.

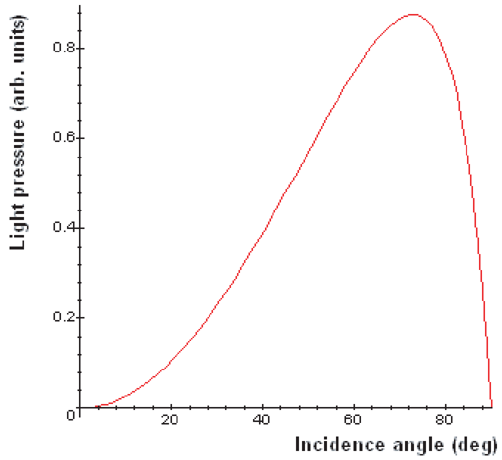
In figures 3 and 4 we represent the dependence of the light pressure at the interface between vacuum and a medium of refractive index n on the angle of incidence and on the refractive index.

It results that at an interface the normal pressure due to light is non-zero only for the p-polarized component of the beam and only for non-zero and non-90° incidence angles. We remind ourselves that we refer only to the light pressure due to beam or propagation medium inhomogeneities.

As is seen from figure 3, the light pressure of the p-component at the interface between vacuum and a medium has a maximum at an angle of incidence of approximately 75°. The reflectivity is zero at the Brewster angle. The light pressure has a maximum at a slightly different value of the incidence angle from the value of the Brewster angle. This is so because of the $\sin^2\theta$ factor. From figure 4 it results that a maximum of the light pressure is acquired for an index of refraction equal approximately to 3, the value of the refractive index corresponding to the maximum being very weakly dependent on the angle of incidence. This type of dependence on the incidence angle and on the refractive index is specific to this type of light pressure given by the gradient force.

There is also a parallel component of the light pressure at the interface. This component is due to the beam intensity distribution across its section and was treated above. An interesting aspect that appears at the interface is that there is a shear stress just at the interface. This is due to the fact that the two media have different refractive indexes. From (7) it results that in medium 1 the lateral (parallel to interface) pressure is

$$p_1 = (1+R)\frac{n_1^2(\lambda) - 1}{2n_1(\lambda)c}[I(r) - I(q)] \quad (22a)$$



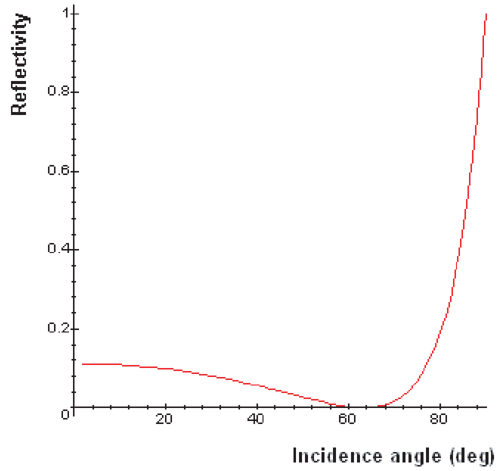


Figure 3. The dependence of the light pressure (top) and of the reflectivity (bottom) of the p-type wave on the incidence angle, constant intensity and refractive index.

while in medium 2 the lateral pressure is

$$p_2 = (1 - R)\frac{n_2^2(\lambda) - 1}{2n_2(\lambda)c}[I(r) - I(q)], \qquad (22b)$$

where n_1, n_2 are the refractive indexes with respect to vacuum. In expression (22*a*) we have considered that both incident and reflected beams contribute to the lateral pressure at the interface, since are two distinct beams. Even if the beam changes its shape at refraction (it changes the shape of its section), its margins coincide when passing from one medium to the other. Because of that we have the same $I(q)$ and $I(r)$ in (22*a*) and (22*b*). The pressure difference σ_{sh} causing the shear stress at the interface is

$$\sigma_{sh} = p_1 - p_2 = \frac{I(r) - I(0)}{2c}\left[\frac{(1 + R)(n_1^2 - 1)g(\theta, \varphi)}{n_1} - \frac{(1 - R)(n_2^2 - 1)g(\theta', \varphi)}{n_2}\right], \qquad (23)$$

where θ is the angle of incidence, θ' is the angle of refraction and φ is the polar angle within the light beam section. $g(\theta, \varphi)$ describes the variation of the beam shape with respect to the angles of incidence, of refraction and of the polar angle. σ_{sh} depends, in general, on the direction along which lateral light pressures are calculated. For normal incidence, $g(\theta, \varphi) = 1$.

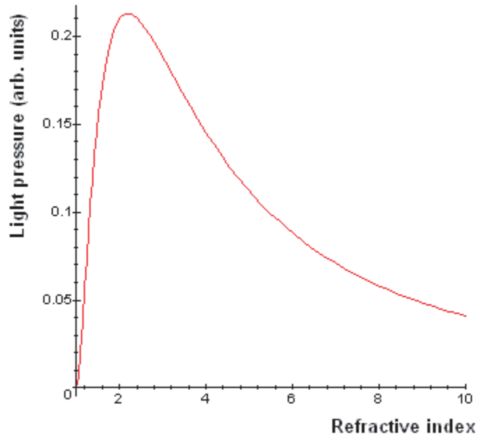


Figure 4. The dependence of the light pressure on the refractive index of the medium, constant angle of incidence and intensity.

As is observed from (23), the shear stress is equal to zero for homogeneous beams (or uniform illumination). This shear stress appears for any kind of beam polarization (s or p) and for any angle of incidence.

This shear stress is another feature of this type of light pressure.

Considering normal incidence and a beam for which $I(q) = 0$, we obtain for $n_1 = 1.5$ and $n_2 = 2$ the following relation for σ_{sh}:

$$\sigma_{sh} = 0.5510^{-9} I(0), \qquad \text{with } \sigma_{sh} \text{ in N m}^{-2} \text{ and } I(0) \text{ in W m}^{-2}.$$

Usually this value for the shear stress is very small and does not present practical implications. If, however, a high power laser beam of a few GW and a section of 1 mm^2 shines on the interface, then the shear stress attains a value of few megapascals, which could change the mechanical behaviour of the materials at the interface and, especially, the behaviour of microstructures.

At the end of this section we make a remark. In the case of lateral pressure inside a medium, the light beam stresses mechanically the medium, which produces an energy transfer from the light beam to the propagation medium during the transient period (of intensity variation with time). This stress consumes some of the light energy and gives rise to an irreversible energy transfer from the light beam to the medium since there are no mechanisms by which the medium gives back to the light beam the energy taken from it. This is true if we think of the case when the beam is intensity modulated. In this case, the pressure is also modulated, giving rise to mechanical waves inside the medium. This wave propagates from the light beam zone towards the margins of the medium, dissipating the energy on the way. By irreversible, we understand the fact that the medium does not give back energy, stored inside it as mechanical energy, to the light beam when the beam exits the medium. The energy loss associated with this type of light pressure will be described in a forthcoming paper [11].

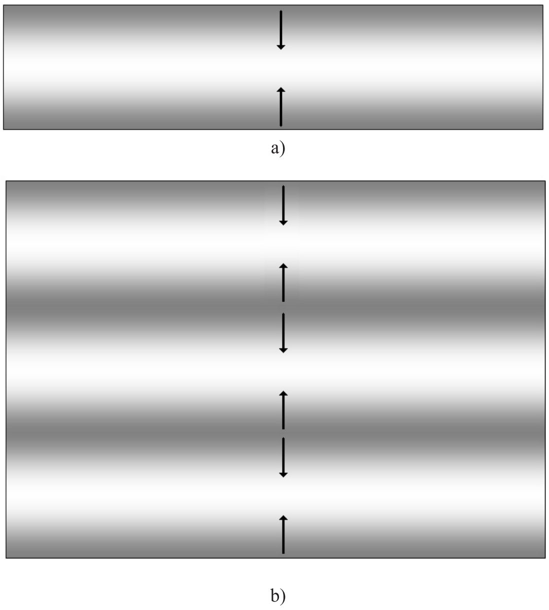


Figure 5. The intensity (coded by shading—see text) and light pressure (represented by arrows) distribution in thin films.

We remind ourselves that all these pressures must be added to those demonstrated in [5] in order to get a picture close to reality as regards the mechanical effect of a light beam on the different types of microstructures.

2.5. *Light pressure inside thin films*

Let us suppose the case of thin film through which a light beam propagates and is incident at an angle θ. In this case, an intensity distribution appears in the thin film [14]. In figure 5, the intensity is encoded by the grey level, the higher intensity having the lighter shade. Because of this non-uniform intensity distribution, the light gradient pressure appears along the thickness of the film and oriented toward the higher intensity regions. The pressure is shown in figure 5 by the black arrow. If the thin film contains only a half of a wavelength, then the situation is depicted in figure 5(a). If the film contains several half-wavelengths, then the situation is depicted in figure 5(b) for the case of three half-wavelengths.

In the first case, there is only a compressive pressure in the thin film. In the second case, the brighter regions are compressed while the darker ones are elongated.

It must be noticed that the pressure depicted in figure 5 appears also when the light intensity across the beam section is uniform. If the beam is non-uniform, then the pressure directed along the film thickness must be added also to that of the lateral component, as described in the preceding subsections. If the beam is incident at a normal angle, then the light propagation direction and the gradient pressure component depicted in figure 5 are parallel. Otherwise, the angle between them is equal to the angle of refraction.

2.6. *Light pressure in the case of polychromatic beams and dispersive medium*

Let us consider that the medium has a slight dispersion and is still transparent or weakly absorbing. We are still in the dispersion regime where equation (1) is applicable. Let us consider also that the incident beam is polychromatic, having a spectral density of intensity equal to $I(\lambda)$. Then, the pressures described by (7) and (19) are obtained by direct integration across the spectrum of the beam and are equal to

$$p_{\mathrm{i}}(q,r) = \int I(\lambda)\frac{n^2(\lambda)-1}{2n(\lambda)c}\,\mathrm{d}\lambda \tag{24}$$

$$p_{\mathrm{v}} = -\frac{\sin^2(\theta)}{2c}(1-R_{\mathrm{p}})\int I(\lambda)\frac{1}{n(\lambda)}\left(\frac{n(\lambda)^2-1}{n(\lambda)^2}\right)^2\mathrm{d}\lambda, \tag{25}$$

with intensity I considered up to now given by the expression:

$$I = \int I(\lambda)\,\mathrm{d}\lambda. \tag{26}$$

We have considered that each spectral component acts independently of the other spectral components and, until now, that we are in the linear regime, i.e. the light gradient pressure does not modify the optical properties of the propagation medium.

2.7. *Light pressure at total internal reflection*

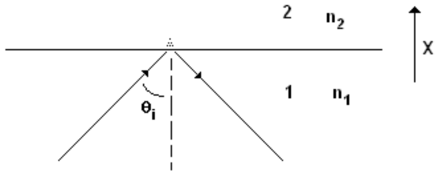


Figure 6. The geometry for total internal reflection.

Consider now that the beam is incident from medium 1 to medium 2, as depicted in figure 6. The two media are semi-infinite. As is known from [14], the light intensity decays exponentially in the less denser medium, according to the expression:

$$I(x) = I_0\exp(-\beta x), \tag{27}$$

where

$$\beta = \frac{4\pi}{\lambda_0}\sqrt{n_1^2\sin^2\theta_{\mathrm{i}} - n_2^2}, \tag{28}$$

where λ_0 is the wavelength in vacuum.

Taking into account equations (8) and (27), we obtain for the light pressure the expression:

$$p = \frac{(n_2^2-1)}{n_2 c}I_0. \tag{29}$$

The pressure appears only in medium 2 and is oriented normal to the surface toward the interface. The pressure is just in the vicinity of the interface. When medium 2 is vacuum the pressure evidently vanishes. An important aspect is that, for semi-infinite media, the pressure does not depend either on the angle of incidence or on the s or p component of the light wave. There is also no dependence on the light wavelength except that due to $n_2(\lambda)$.

Let now consider the case of frustrated internal reflection (see figure 7). In this case medium 2 has a thickness d and is placed between two semi-infinite halves of medium 1.

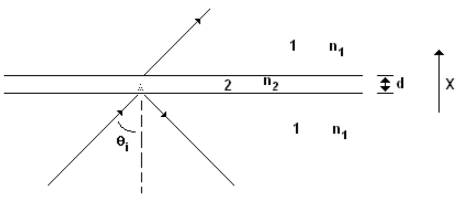


Figure 7. The geometry for the frustrated internal reflection.

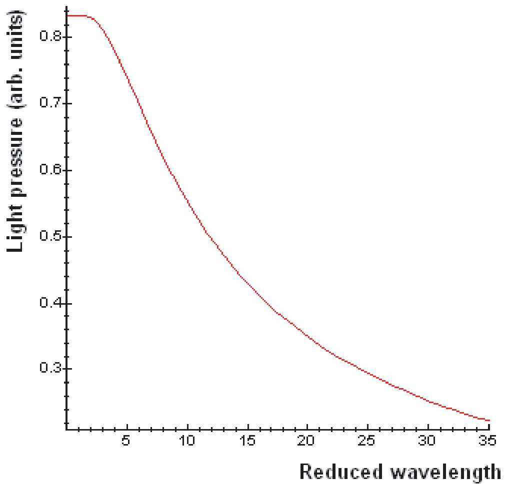


Figure 8. The dependence of the light pressure on the reduced wavelength for the case of frustrated internal reflection, constant intensity.

In this case, by integrating the density of force between the boundaries of medium 2 (where the intensity is non-uniform), we obtain

$$p = \frac{(n_2^2 - 1)}{n_2 c} I_0 (1 - \exp(-\beta d)). \tag{30}$$

The pressure is directed again toward the lower half of medium 1, but is smaller as compared to the case of total internal reflection. Moreover, it depends on the angle of incidence (through β). It also depends strongly on the wavelength (through β), even if the two media are practically non-dispersive. In figure 8 is depicted the dependence of the light pressure on the reduced wavelength (that is, on the wavelength λ_0 in vacuum divided by the distance d).

Another difference between the pressure in the case of total internal reflection and the pressure in the case of frustrated internal reflection is in the dependence on the refraction index. Thus, in the case of total internal reflection the light pressure increases with the refractive index of medium 2. In the case of frustrated internal reflection the pressure first increases with n_2 and reaches a maximum (at a value of n_2 depending on n_1), after which it decreases up to zero for $n_2 = n_1 \sin\theta_i$.

The overall effect in the case of frustrated internal reflection is that the two halves of medium 1 are getting closer as the consequence of the light pressure acting on medium 2, as if there is an attractive force between them. I point out the fact that the two halves of medium 1 are semi-infinite is crucial. If they are finite and allow a mode structure inside them (like a waveguide), then the force between them could be either attractive or repulsive, depending on the symmetric/antisymmetric structure of the modes in the two waveguides, as described in [12]. If the two halves are semi-infinite, then there is only attraction between them. More precisely, they behave as if there is an attraction, the effect being due to the compression of medium 2 toward the lower half of medium 1 because of the light pressure and not of a net force acting on the two halves. I mention that at the interface between medium 2 and the upper half of medium I there is no pressure, since the 'angle of incidence' at that interface is 90°. According to equations (18) and (19) and to figure 3, the pressure in this case is null.

3. Other sources of force in non-homogeneous intensity light fields

Up to now we have considered only the contribution of dipoles, through the resulting polarization **P**, to the force, respectively pressure, exerted in a medium by a light beam with an inhomogeneous light intensity distribution. In some cases, however, the medium also may contain higher-order multipoles such as quadrupoles, octupoles, etc. These multipoles contribute too to the net force exerted by the inhomogeneous electric field. We shall mention briefly the expression of force in the case of quadrupoles. In the approximation that the electric field does not change the quadrupolar momentum and does not induce a dipole moment of the quadrupole (linear approximation), the force $\mathbf{f}_\alpha$ acting on the quadrupole of quadrupolar momentum **Q** is given by [13]

$$\vec{f}_\alpha = \tfrac{1}{6} Q_{\beta\gamma} \nabla_\alpha (\nabla_\beta E_\gamma), \tag{31}$$

where $Q_{\beta\gamma}$ is the component of the quadrupole momentum tensor **Q**. As is observed from equation (31), the force depends on $\nabla^2\mathbf{E}$. It is null for a homogeneous beam or for a beam with a linear varying intensity distribution.

The force **f** per unit volume is obtained by summing the force given by equation (31) over all the N quadrupoles contained in the unit volume:

$$\vec{f} = \sum_{\alpha}^{N} \vec{f}_\alpha. \tag{32}$$

The light pressure p_q is directly obtained from equation (32) by straightforward integration:

$$p_q = \int_{x_i=0}^{x_i R} \vec{f}(x_i)\,\mathrm{d}x_i. \tag{33}$$

This pressure adds to the conventional light pressure and to that due to the gradient force.

For making correct estimations of the mechanical effect of light on different bodies and interfaces all the light pressure components should be considered. Inside a body, only the gradient force and, more generally, multipolar components give a pressure if the light intensity distribution is non-uniform.

4. Comparison between the two types of light pressure

The light pressure considered in this paper originates from the non-uniform light electric field distribution and is due to the force acting on the dipole and multipole moments of the material when such a non-uniform field is applied.

As results from (7), this kind of light pressure has some specific features, such as:

- it is transversal to the propagation direction, not only longitudinal, whatever the polarization of the beam (s or p). In the case of [5] the lateral pressure appears compressive for s-polarized beams while for p-polarized beams it is extensive. In our case, the sign of the pressure is given only by the light intensity difference between the two considered points and does not depend on polarization. As compared to the usual light pressure, the sign of the pressure is given by the light distribution across the beam section, i.e. it appears also inside the beam section and not only at the edges.
- the lateral (gradient) pressure demonstrated by us appears also at normal incidence, in the case of [5] the lateral pressure vanishes at normal incidence. There is no need, as in [5], that there be several beams that interfere.
- its value along certain directions (perpendicular to the beam propagation direction) depends on the mode structure of the light beam, because the intensity depends in general on both the distance from the centre of the beam and on the angle φ. Because of that, the integrand in (7) depends on the mode structure and so does the pressure.
- it vanishes for those cases when the transparent material is entirely illuminated with a light beam of uniform intensity. Otherwise, in the case of absorption, a pressure directed along the propagation direction and oriented toward the higher intensity region appears.
- its direction is always the same at the edge of the beam, where $I(q) = 0$.
- in general, at the interface, reflectivity does not contribute (excepting the loss of power) at the normal (perpendicular) gradient pressure. Only the transmitted beam gives light pressure. For the conventional light pressure described in [5], reflectivity adds pressure at the interface.
- at the interface, the s-polarized beams give no normal (perpendicular) pressure, whatever the angle of incidence. In the case of conventional light pressure, the s-polarized beam exerts always a normal pressure at the interface.
- in the case of a p-polarized beam incident on the interface, only its vertical component gives a normal light pressure and only at non-zero incidence angles. The dependence on the refractive index and on the incidence angle is entirely different as compared to the usual light pressure. In both cases the light pressure due to the gradient force shows a maximum at certain values of the refractive index (around 3) and, respectively at an angle of incidence slightly larger than the Brewster angle.
- a shear stress develops at the interface due to the difference in lateral light pressures acting in the two media above and below the interface. This effect is maximum at normal incidence.
- in thin films there is also a gradient pressure component directed along the film thickness (and parallel to the propagation direction for normal incidence) and directed towards higher intensity regions. This pressure appears even for uniform intensity light beams.
- at frustrated internal reflection the light pressure also shows a maximum with respect to the refractive index of the less denser medium and the semi-infinite halves of the optically denser medium are always attracting each other (more precisely, the less denser medium is compressed and the two halves are getting closer as if there is attraction).

5. Discussions

We have supposed that the light pressure does neither change the shape of the propagation medium nor stresses it so that nonlinear effects appear. However, if the medium is easily deformable (compressible) then, in the case of a Gaussian beam, we expect that light will be confined to the beam centre. We argue that by considering that the light pressure compresses the propagation medium to the beam axis. Usually, this compression produces a slight increase of the refractive index at the centre of the beam, focusing of the beam thus taking place. In this way, the beam may self-focus by mechanical means rather than by a nonlinear optical response of the propagation medium. However, the beam intensity for producing such refractive index changes is quite high and optical nonlinear effects (at least of the Kerr type) may take place prior to this optomechanical effect nonlinearity. This nonlinear optomechanical interaction needs a separate consideration and will be not presented here.

Another aspect regarding the light pressure produced by the gradient force is the presence of the pressure component at twice the light beam frequency (see equation (5), the f_{opt} component). Up to now I have considered that this term is irrelevant. This assumption is valid as long as the corresponding wavelength of the mechanical oscillation at that frequency is less than twice the lattice constant of the medium (in the case of crystalline solids) or less than some specific length of the medium (for example, a multiple of the mean free path) in gases and liquids, since in these conditions it cannot propagate. I will make some remarks only for the case of crystalline solids. The assumption considered above is valid at optical frequencies. For a beam in the microwave and far-infrared regions, the assumption is no longer true. In this case, the f_{opt} component of the light pressure given by equation (5) manifests also in the medium. We must recall that there is a pressure wave at twice the frequency of the optical beam. This mechanical wave, having a wavelength several times larger than the lattice constant, mixes with the phonons of the solid, phonons that are present at the respective frequencies, since the mechanical wave induces oscillations of the crystalline lattice. The phonons are also oscillations of the lattice. The effect of this light pressure component is the heating of the crystal, since it increases the phonon population at its frequency. Detailed calculations need further study and are the subject of another paper. The main point is that the solid heats up as a consequence of the light pressure, respectively as a consequence of non-uniform light intensity distribution. A uniform intensity distribution does not heat the solid by this mechanism, since the respective light pressure component is equal to zero. To my knowledge, this heating mechanism due to the non-uniform light intensity distribution of microwave and far-infrared beams is novel and was not

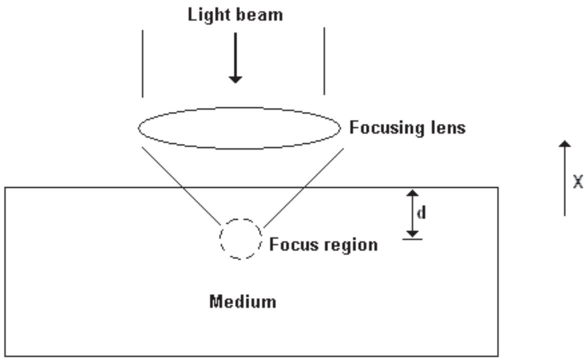


Figure 9. Schematic diagram of localized pressure generation inside a medium by using light pressure.

reported previously. The magnitude of the pressure amplitude (and hence of the heating) depends on the concrete intensity distribution, since its magnitude is given by an integral of the form

$$p_{\mathrm{opt}} = \int_0^q f_{\mathrm{opt}} \cos(2kx)\,\mathrm{d}x \approx \int_0^q \mathrm{grad}\,[I(x)] \cos(2kx)\,\mathrm{d}x. \quad (34)$$

The value of the pressure amplitude does not exceed the value of the static (low frequency) component, so that I do not expect a significant heating of the solid for beams of moderate intensity.

As consequences of the static (low frequency) light pressure due to the gradient force, I mention that the effect of this type of light pressure is as important for microsystems as that of conventional light pressure. I mention also the fact that a beam modulated in intensity produces a modulated pressure inside a medium, which means it generates sound. This aspect is considered in forthcoming papers [15, 16] and may find practical applications in material and microsystems characterization, as well as in exciting microsystems based on thin films. On the other hand, the sound produced by the pulse, if detected by a sensitive microphone, could allow the detection of light without absorbing its intensity. Of course, the detection efficiency is low, but still usable [15].

Another interesting application is the possibility to apply pressure in a small region inside a medium, at a desired point. For example, let us consider a focused beam inside a medium, as depicted in figure 9.

By having a variable zoom, the focal region at distance d from the surface can be moved along the X axis at the desired speed. When this speed exceeds the speed of sound in the medium, a shock wave develops, as in the case of supersonic movement. Moreover, if the light intensity is temporally modulated, vibrations are generated in the material in the desired region(s). This technique could be useful in material characterization.

A special discussion has to be made as regards the computations made at the interface between two media. I must state here that I have considered an infinitely thin interface layer. Let $\varepsilon(x)$ be the permittivity in the interface layer, varying with x coordinates. Since this is the integration of a gradient function from point 1 in medium 1 to point 2 in medium 2 (see equations (13) and (17)), we obtain the value of ε in these two points, respectively. Points 1 and 2 can be put as close as possible to the interface but not actually on the interface. In fact, the notion of an interface at such small distances loses its significance. All the papers dealing with macroscopical electrodynamics consider the approximation of continuous medium. When the points are very close to the interface, the continuous medium approximation is no longer valid: the discrete structure of the medium must be considered. For example, when calculating the image force in this case we must consider the discrete/atomistic structure of the medium (see [17]) and do not use the continuous medium approximation that leads us to singularities at the interface. On the other hand, the interface thickness is far less than the radiation wavelength, so fixing the limits of integration in medium 1 and, respectively, in medium 2 (and very close to the interface) is correct from the macroscopic point of view. Considering the interface layer that changes from medium 1 to medium 2 needs an atomistic approach and the calculations have to be changed.

The atomistic treatment takes into account the concrete discrete structure of the interface and could lead to results dependent on the specific material. In this case, we have a three-layer problem: medium 1–interface–medium 2. The atomistic approach should give us the law of variation of $\varepsilon(x)$, from which we can make all the integrations easily in the three-layer model. This is the correct approach, as mentioned also by [6], and not that of averaging ε which is done by different authors. But since this space region where we have an $\varepsilon(x)$ dependence is of the order of several angströms, the continuous medium approximation and the macroscopic approach give us a good result. The definition of the interface as a standalone layer is extremely important at non-normal incidence also for the usual light pressure, since the angle of the Poynting vector in the layer (and hence of the usual light pressure) will depend strongly on $\varepsilon(x)$. However, as we said above, the interaction of light with the interface must be considered for each atomic layer and the macroscopic refraction law in the case of the very thin interface should be replaced by the form resulting from the atomistic description.

6. Conclusion

I have described theoretically the behaviour of the light gradient pressure due to the non-uniform distribution intensity across a light beam section. I have deduced the corresponding expressions and main features, both when the beam is entirely contained in a medium and at the interface between two different media. The results are important both for the field of optics (see [11]) and for the field of microsystems and nanosystems. The magnitude of this kind of light pressure is of the same order as that of conventional light pressure, which means that its effect may not be neglected. Moreover, there is also a harmonic component of the pressure for microwave and far-infrared beams, a component that heats up the propagation medium. Discussions regarding some possible applications, as well as comparison to the usual light pressure, are also presented.

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References

[1] Lebedev P N 1901 Experimental examination of light pressure *Ann. Phys., Lpz.* **6** 433

[2] Shen F, Xu J and Wang A 2005 Measurement of the frequency response of a diaphragm-based pressure sensor by use of a pulsed excimer laser *Opt. Lett.* **30** 1935

[3] Dragoman D and Dragoman M 2001 Characterization of wavefront of light beams by use of tunnelling cantilevers *Appl. Opt.* **40** 678–82

[4] Paternostro M, Gigan S, Kim M S, Blaser F, Böhm H R and Aspelmeyer M 2006 Reconstructing the dynamics of a movable mirror in a detuned optical cavity *New J. Phys.* **8** 107

[5] Mansuripur M 2004 Radiation pressure and the linear momentum of the electromagnetic field *Opt. Express* **12** 5375

[6] Barnett S M and Loudon R 2006 On the electrodynamic force on a dielectric medium *J. Phys. B: At. Mol. Opt. Phys.* **39** S671–84

[7] Gordon J P 1973 Radiation forces and momenta in dielectric media *Phys. Rev.* A **8** 14

[8] Landau L D, Lifshitz E M and Pitaevskii L P 1984 *Electrodynamics of Continuous Media* (*Landau and Lifshitz Course in Theoretical Physics*) 2nd edn vol 8 (Oxford: Pergamon)

[9] Matveev A N 1986 *Electricity and Magnetism* (Moscow: Mir) (Engl. Transl.)

[10] Born M and Wolf E 1999 *Principles of Optics* 7th edn (Cambridge: Cambridge University Press)

[11] Antonoyiannakis M I and Pendry J B 1999 Electromagnetic forces in photonic crystals *Phys. Rev.* B **60** 2363–74

[12] Povinelli M, Lončar M, Ibanescu M, Smythe E J, Johnson S G, Capasso F and Joannopoulos J D 2005 Evanescent-wave bonding between optical waveguides *Opt. Lett.* **30** 3042–4

[13] Jackson J D 1974 *Classical Electrodynamics* 2nd edn, vol 1 (New York: Wiley)

[14] Born M and Wolf E 1999 *Principles of Optics* 7th edn (Cambridge: Cambridge University Press)

[15] Moagăr-Poladian G 2007 Light pressure based method for measuring refractive index *Opt. Lett.* to be submitted

[16] Moagăr-Poladian G 2007 The optical rectification pseudo-effect *Opt. Lett.* submitted

[17] Desjonqueres M C and Spanjaard D 1996 *Concepts in Surface Physics* (Berlin: Springer)