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Constructing Non-Periodic Tiling Patterns with P System

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Abstract. Parametric Tile Pasting P System (PTPPS) is a new computational model, based on the structure and functioning of living cells, to generate tiling patterns, using simple rules of gluing tiles (pasting rule) at their edges and geometric operations on the tiles. In this paper, a variant of PTPPS, namely Tissue-like Parametric Tile Pasting P System, with new geometric operations, to generate non-periodic patterns is introduced. The computational complexity of the system and the power of the system with different geometric operation are also discussed.

Key-words: P systems, Pasting system, Two dimensional pattern.

1. Introduction

Tilling or tessellation is a branch of combinatorial geometry concerned with the ways shapes fit together to fill the Euclidean plane without gaps or overlaps. In the literature of tiling theory, various methods have been studied for the generation of aperiodic and periodic tiling patterns. H. Wang [9] generated a tiling of the plane using the set of square tiles with marked edges with a condition that the edges of the tiles should be of same colour. C. Goodman Strauss [4] has constructed aperiodic hierarchical tiling using substitution rules. De Brujin [2] has discovered a method to generate non-periodic patterns using canonical substitution rules. Recently T. Robinson et al. [14,16], has constructed a new method using pasting rules to generate tiling and tessellation patterns. In the following we recollect the notion of Membrane Computing and systems which uses pasting rules to generate two dimensional patterns.

Membrane Computing (or P system) [7, 11] is an area of computer science aiming to abstract computing models from the structure and functioning of living cells. A P system consists of a membrane structure, multisets of objects placed inside the membranes, evolution rules governing the modification of these objects in time, and transfer of objects from one membrane to another membrane (inter-cellular communication). In recent years, generation of two dimensional picture languages using P systems has extensively studied. P systems generating arrays and Indian folk designs (kolam patterns) have been considered in [18]. Tissue like P systems with active membranes that generate local picture languages have been considered in [1].

Recently, a new bio-inspired computational model, namely Tile Pasting P system and its variants, based on the structure and functioning of living cells have been studied in [5, 14, 19] for generating two dimensional tiling and tessellation patterns, that were formed by gluing (pasting rule) regular polygon tiles, thereby covering the infinite Euclidean plane without gaps or overlaps. In [5] a variant of Tile Pasting P System, namely, Parametric Tile Pasting P system was introduced with geometric operations on the tiles. In this paper, a variant of PTPPS, namely Tissue-like Parametric Tile Pasting P System (t-PTPPS), with new geometric operations, to generate non-periodic patterns has been considered. We demonstrated the generative capabilities of the system with different geometric operations by considering the alternative boat sequence, self similar tiling pattern and chair like tiling pattern (Examples 1, 2 and 3 respectively). The computational complexity of the system and the power of the system with different geometric operation have also been discussed.

2. Tissue Like Parametric Tile Pasting P System

Tissue-like Parametric Tile Pasting P System (t-PTPPS), introduced in [6] is a new tile pasting system with tissue like membrane structure. In this type of system, the membranes several of them freely placed in a environment, processes the tiles in each region using pasting rules and geometric operations and communicates the tiling patterns to other membranes of the system. This model is found to generate periodic as well as non-periodic patterns. In this section examples of t-PTPPS generating non periodic patterns with various geometric operations are presented.

In the following the notion of pasting rule for tiles and the geometric operations required for the generation of non-periodic tiling patterns are presented.

A pasting rule [16,19] is a pair (x, y) of edge labels of two (not necessarily different) square tiles a and b. Again x and y need not be different. If x is the label of the right (respectively left) edge of a and y is the label of the left (respectively right) edge of b, then an application of the rule (x, y) pastes side by side (or joins edge to edge) the two tiles a and b. We can likewise define pasting of two tiles one above the other. Tiling patterns are thus made up of tiles, 'glued' (or pasted edge to edge) together.

Given an axiom tile (proto tile) or an arbitrary tile t_i with label a, denoted by (t_i, a), the following operations are considered.

1. Rotation: The operation rotation on tiles and tiling patterns is the process of repositioning it along a circular path in the xy plane. To rotate a tile or a tiling pattern (t_i, a), we specify a rotation angle θ and the position (x, y) of the rotation point (pivot point) about which the tile is to be rotated. Positive values for the rotation angle define counterclockwise rotations about the pivot point and negative values rotate tiles in the clockwise direction. The resultant tiling pattern t_j is labeled as b.

$$[_l(t_i, a)]_l \rightarrow_{(\theta,x,y)} [_m(t_j, b)]_m$$

2. Edge/tile relabeling: By this operation the edges of the tiles and tiling patterns (with the tile label a) (t_i, a), are relabeled from the edge relabeling set μ and the resultant tiles and tiling patterns (labeled as b) (t_j, b), is communicated to the membrane m. The operation of pasting of edges has preference over the operation of edge relabeling, if both the operation are present in an evolution set. When $\mu = \phi$, the tile t_i with label a is redefined as tile t_j with label b and communicated to the membrane m.

$$[_l(t_i, a)]_l \rightarrow_{\mu} [_m(t_j, b)]_m$$

3. Path rewriting (vertex removal and merging of edges): By this operation a path bounding a tile or tiling is rewritten (removing one or more vertices and merging edges of the path) by a new path of vertices without altering the shape of the tile. For instance a rule of the form $v_i(e_i v_{i+1} e_{i+1}) v_{i+2} \dots e_{i+k} v_{i+n} \rightarrow v_i(e_i) v_{i+2} \dots e_{i+k} v_{i+m}$ removes the vertex v_{i+1}, merges the edges e_i and e_{i+1} and rewrites the resultant edge between v_i and v_{i+2} as e_i. The application of path rewriting rules to a pattern has preference over communication of patterns from one membrane to another membrane.

4. Scaling and Edge relabeling: The operation of scaling on a tile alters the size of the tile. The scaling factor $\{s_x, f(n_1), \mu\}$ and $\{s_y, f(n_2), \mu\}$ scales the tile in the x and y direction respectively according to the function $f(n_i)$ and the edges of the resultant tiling are relabeled using the set μ. The scaling factor $\{s_{(x,y)}, f(n_1), f(n_2), \mu\}$ scales the tile in the x and y direction simultaneously according to the function $f(n_1)$ and $f(n_2)$ respectively. A uniform scaling of the tiles is produced if $f(n_1) = f(n_2)$. After the scaling operation on the tile, the edges of the tile are relabeled. Thus we get a new tile after the scaling and edge relabeling operation.

$$[_l(t_i, a)]_l \rightarrow_{(s_x, f(n), \mu)} [_m(t_j, b)]_m$$

where $f(n)$ is the scaling function of the edge and μ is the edge relabeling set.

Formally a Tissue-like Parametric Tile Pasting P System is a construct of

$$\Pi = (O, T, M, (t_0)_i, \Sigma, Syn, T_i, R_{(i,j)}, i_0), \quad i, j \in (1, 2, \dots, m)$$

where

- O is an alphabet of labels of the edges of the tiles.
- $T \subseteq O$ is the terminal alphabet.
- M is a finite set of membranes with labels from the set $\{1, \ldots, m\}$.
- $(t_0)_i$, $i \in \{1, \ldots, m\}$ is the axiom tile present in the membrane m_i.
- Σ is the finite set of tiles associated with the region of the membranes m_i, $i \in \{1, \ldots, m\}$.
- $Syn = \{(i, j)/i, j \in \{1, \ldots, m\}\}$ is the set of links among the membranes (synapses).
- T_i, $i \in \{1, \ldots, m\}$, are the finite set of tables, containing sets of the form $\{R_{(i,j)}/(i, j) \in syn, i, j \in (1, \ldots, m)\}$.
- $R_{(i,j)}$ is a finite set containing evolution rules. The evolution rules can be defined in any one of the following form or a combination of these forms.

  a) (x, y), where $x, y \in O$, concerned with a pair of edges of equal size, which allows the edges (commutatively) of the corresponding tiles to get glued (pasting rule).

  b) $[_i(t_i, a)]_i \rightarrow_X [_j(t_j, b)]_j$, where X is any of the operations, namely, rotation, edge relabeling, scaling and edge relabeling, path rewriting operations for a tiling pattern (t_i, a) in the membrane m_i. The resultant tiling pattern (t_j, b) is communicated to the membrane m_j.

  c) $[_l(t_i, a)]_l \rightarrow [_m(t_i, a)]_m$, here the application of the rule communicates the pattern t_i from membrane l to the membrane m.

- i_0 is the output membrane.

The computation starts with the membrane m_i, containing the axiom tile $(t_0)_i$. During the application of the evolution rules in the set $R_{(i,j)}$, pasting operation between tiles in the region m_i has preference over communication of tiling patterns between the membranes m_i and m_j. Also if a membrane m_i contains two tables T_i, then any one of them is selected non-deterministically. A sequence of transitions forms a computation and the result of a halting computation is the set of patterns over Σ sent to the output membrane during the computation. During the computation of patterns, the evolution sets $R_{(i,j)}$ present in the table T_i are applied either one by one non-deterministically or all the sets are applied simultaneously. A system in which the evolution sets $R_{(i,j)}$, from the table T_i, are applied one by one non-deterministically is called Maximally parallel. For such a system in each time unit, if T_i contains two or more evolution sets $R_{(i,j)}$, then any one of the set $R_{(i,j)}$ is chosen non-deterministically and the evolution rules present in the set $R_{(i,j)}$ are applied simultaneously to all the edges of the tiling present in the membrane region.

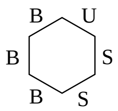
A system in which the evolution sets $R_{(i,j)}$, from the table T_i, are applied simultaneously is called Completely Maximally parallel. For such a system in each time unit, if T_i contains two or more evolution sets $R_{(i,j)}$, then all the sets $R_{(i,j)}$ are applied and the evolution rules present in the set $R_{(i,j)}$ are also applied in a parallel manner in a region of membrane m_i, containing the pattern t_i.

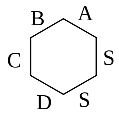
A system is said to be non-extended if $O = T$. In the case of all non-extended systems, all tilings sent to the output membrane are accepted. A tiling pattern which remains in another membrane than i_0 or in the case of extended system, which reaches the output membrane but contains labels not in T does not contribute to the generated language. The set of all such picture patterns computed or generated in this way by a t-PTPPS Π is denoted by $PL - PTPPS(\Pi)$. The language generated by systems Π as above, with at most m membranes, and geometric conditions $\alpha, \beta \in$ {rotation, edge or tile relabeling, path rewriting, scaling and edge relabeling}, is denoted by $PL_m(t\text{-}PTPPS, \alpha, \beta)$.

Example 1. Consider a non-extended, maximally parallel, tissue-like Parametric tile Pasting P system $\Pi \in PL_4(t\text{-}PTPPS, \alpha, \phi)$ with geometric operations α = edge relabeling.

$$\Pi = (O, T, M, (t_0)_1, \Sigma, Syn, T_i, R_{(i,j)}, m_4)$$

where

- $O = \{A, B, C, D, S, U, L\}$.
- $T = O$.
- M is a set of 4 membranes with labels from the set $\{1, \ldots, 4\}$.
- $(t_0)_1 =$ 

is the axiom tile present in the membrane m_1.
- $Syn = \{(1,2), (2,3), (2,4), (3,2), (3,4)\}$.
- $\Sigma = \Bigg\{$ 

$\Bigg\}$.
- $T_1, \ldots, T_4$ are the finite set of tables, containing the evolution rules $\{R_{(i,j)}/(i,j) \in Syn, i, j \in (1, \ldots, 4)\}$.
  $T_1 = \{R_{(1,2)}\}$, $T_2 = \{R_{(2,3)}, R_{(2,4)}\}$, $T_3 = \{R_{(3,4)}, R_{(3,2)}\}$, $T_4 = \phi$
- $R_{(1,2)} = \{(A, B), [_1 t_i]_1 \to [_2 t_i]_2\}$
  $R_{(2,3)} = \{(A, B), [_2 t_i]_2 \to [_3 t_i]_3, [_2 t_i]_2 \to \mu[_3 t_j]_3$, where $\mu_1 = (S \to L)\}$
  $R_{(2,4)} = \{(A, B), [_2 t_i]_2 \to [_4 t_i]_4\}$
  $R_{(3,4)} = \{(B, L), [_3 t_i]_3 \to [_4 t_i]_4\}$
  $R_{(3,2)} = \{(B, L), [_3 t_i]_3 \to \mu_1[_2 t_{i+1}]_2$, where $\mu_2 = (C \to B, D \to B)$
  $[_3 t_i]_3 \to \mu_2[_2 t_j]_2$, where $\mu_3 = (U \to A, L \to A)\}$

- m_4 is the output membrane.

The first four members of alternate boat sequence obtained in the output membrane is shown in the Fig. 1.

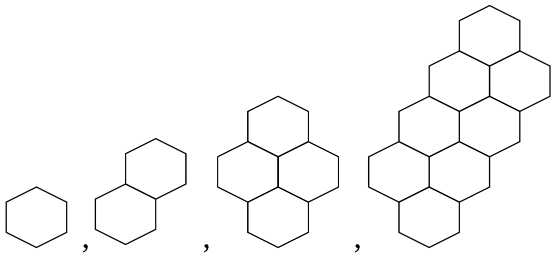
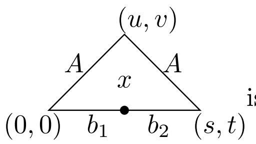


Fig. 1. Alternate Boat Sequence.

Example 2. Consider the extended, completely maximally parallel, tissue-like Parametric Tile Pasting P System $\Pi \in PL_{15}(t\text{-}PTPPS, \alpha, \beta)$ where α = rotation and β = scaling and edge relabeling operation.

$$\Pi = (O, T, M, (t_0)_1, \Sigma, Syn, T_i, R_{(i,j)}, m_{15}), \quad i, j \in (1, \dots, 15)$$

where

- $O = \{A, a, b_1, b_2\}$.
- $T = \{A, b_1, b_2\}$.
- M is a set of 15 membranes with labels from the set $\{1, \dots, 15\}$.
- $(t_0)_1 =$ 

is the axiom tile present in the membrane m_1.
- $\Sigma = \{\phi\}$.
- $Syn = \{(1,2), (1,3), (2,4), (3,4), (3,5), (4,6), (5,6), (5,7), (6,8), (7,9), (7,8), (8,10), (9,10), (9,11), (10,12), (11,13), (12,14), (13,14), (14,15), (14,2), (14,3)\}$ is the set of links among the membranes.
- $T_1 = \{R_{(1,2)}, R_{(1,3)}\}$, $T_2 = \{R_{(2,4)}\}$,
  $T_3 = \{R_{(3,4)}, R_{(3,5)}\}$, $T_4 = \{R_{(4,6)}\}$,
  $T_5 = \{R_{(5,7)}, R_{(5,6)}\}$, $T_6 = \{R_{(6,8)}\}$,
  $T_7 = \{R_{(7,9)}, R_{(7,8)}\}$, $T_8 = \{R_{(8,10)}\}$,
  $T_9 = \{R_{(9,10)}, R_{(9,11)}\}$, $T_{10} = \{R_{(10,12)}\}$,
  $T_{11} = \{R_{(11,13)}\}$, $T_{12} = \{R_{(12,14)}\}$,
  $T_{13} = \{R_{(13,14)}\}$, $T_{14} = \{R_{(14,2)}, R_{(14,3)}\}$,
  $T_{14} = \{R_{(14,15)}\}$, $T_{15} = \{\phi\}$.

- The evolution rule sets for the generation of the self similar tiling with triangle boundary is given below.
$R_{(1,2)} = \{[_1(t_0, x)]_1 \to [_2(t_0, x)]_2\}$
$R_{(1,3)} = \{[_1(t_0, x)]_1 \to_{(s_{x,y}, 1/\sqrt{2}, \mu)} [_3(t_i, x)]_3\}$ where $\mu = \{A \to a\}$
$R_{(2,4)} = \{[_1(t_0, x)]_2 \to [_4(t_0, x)]_4\}$
$R_{(3,4)} = \{[_3(t_i, x)]_3 \to_{[45,(0,0)]} [_4(t_j, x)]_4, [_3(t_i, x)]_3 \to_{[315,(0,0)]} [_4(t_j, x)]_4\}$
$R_{(3,5)} = \{[_3(t_i, x)]_3 \to_{(s_{x,y}, 1/\sqrt{2}, \phi)} [_5(t_j, x)]_5\}$
$R_{(4,6)} = \{(b_1, a), (b_2, a), (a, a), [_4 t_i]_4 \to [_6 t_j]_6\}$
$R_{(5,6)} = \{[_5(t_i, x)]_5 \to_{[90,(0,0)]} [_6(t_j, x)]_6$
$[_5(t_i, x)]_5 \to_{[270,(0,0)]} [_6(t_j, x)]_6$
$[_5(t_i, x)]_5 \to [_6(t_j, x)]_6\}$
$R_{(5,7)} = \{[_5(t_i, x)]_5 \xrightarrow{\mu} [_7(t_j, x)]_7\}$, where $\mu = \{a \to A\}$
$R_{(6,8)} = \{(b_1, a), (b_2, a), (a, a), [_6 t_i]_6 \to [_8 t_i]_8\}$
$R_{(7,8)} = \{[_7 t_i]_7 \to_{[90,(0,0)]} [_8 t_j]_8$
$[_7 t_i]_7 \to_{[270,(0,0)]} [_8 t_j]_8\}$
$R_{(7,9)} = \{[_7 t_i]_7 \to [_9 t_i]_9\}$
$R_{(8,10)} = \{(b_1, b_2)[_8 t_i]_8 \to [_{10} t_i]_{10}\}$
$R_{(9,10)} = \{[_9 t_i]_9 \to [_{10} t_i]_{10}\}$
$R_{(9,11)} = \{[_9 t_i]_9 \to [_{11} t_i]_{11}\}$
$R_{(10,12)} = \{(A, A), [_{10} t_i]_{10} \to [_{12} t_i]_{12}\}$
$R_{(11,13)} = \{[_{11} t_i]_{11} \to [_{13} t_i]_{13}\}$
$R_{(12,14)} = \{[_{12} t_i]_{12} \to [_{14} t_i]_{14}\}$
$R_{(13,14)} = \{[_{13}(t_i, x)]_{13} \to [_{14}(t_i, x)]_{14}\}$
$R_{(14,3)} = \{[_{14}(t_i, x)]_{14} \to_{(s_{x,y}, 1/\sqrt{2}, \mu)} [_3(t_i, x)]_3\}$ where $\mu = \{A \to a\}$
$R_{(14,2)} = \{[_{14} t_i]_{14} \to [_2 t_i]_2\}$
$R_{(14,15)} = \{[_{14} t_i]_{14} \to [_{15} t_i]_{15}, [_{14}(t_i, x)]_{14} \to [_{15}(t_i, x)]_{15}\}$
- m_{15} is the output membrane.

We apply the completely maximally parallel method discussed in this section for the generation of self similar tiling with triangle boundary. Self similar tilings are tilings in which the tiles reduce in size in a proportinate manner in the tiling [13]. The tilings are constructed by reducing the prototiles by a scaling factor, defined by a function, and then matching the long edges of the next smaller generation to the short edges of the larger generation. The tiling is generated by pasting the scaled down proto tiles row by row successively.

For each row of the tiling, we require edge relabelled and scaled down proto tiles rotated to an angle of 45°, 90°, 270° and 315° supplied to the membrane regions m_3, m_5, m_7 and m_9 in a particular order. The rotated and scaled down proto tiles are pasted in a row successively by the pasting rules and evolution rules defined in the membranes m_4, m_6, m_8 and m_{10}.

The membrane m_{14} has two evolution sets T_{14} containing evolution rules. During computation any one of the table is selected non-deterministically and the evolution rules present in the table has been applied. After completing each row of the tiling the resultant pattern is communicated back to the system for the next level of generation if the table containing the evolution rule set $R_{(14,2)}$ and $R_{(14,3)}$ are applied. The computation stops when the table containing the evolution rule set $R_{(14,15)}$ is applied and the tiling pattern is collected in the output membrane m_{15}.

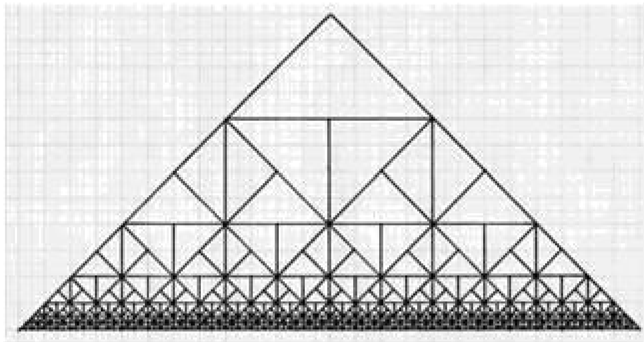


Fig. 2. A self similar tiling with Triangle boundary.

The communication graph of tissue-like parametric tile pasting P system generating the self similar patterns is shown below.

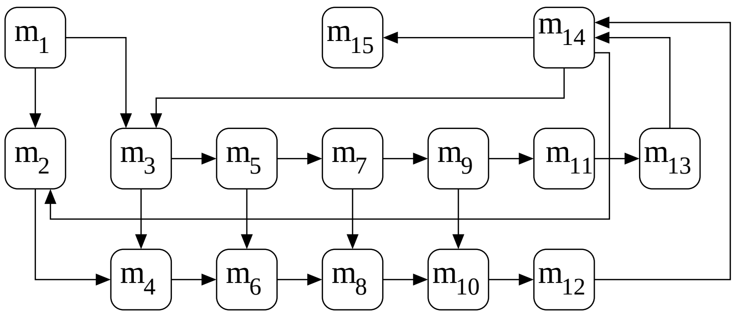


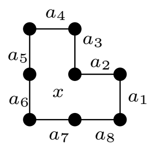
Fig. 3. Communication graph of the system.

Example 3. Consider the extended, maximally parallel, tissue-like Parametric Tile Pasting P System $\Pi \in PL_5(t\text{-}PTPPS, \alpha, \beta)$ where α = rotation and β = path rewriting operation.

$$\Pi = (O, T, M, (t_0)_1, \Sigma, Syn, T_i, R_{(i,j)}, m_5), \quad i, j \in (1, \ldots, 5)$$

where

- $O = \{a_i, b_i, c_i, d_i \text{ for } 1 \leq i \leq 8\}$.
- $T = \{a_i, \text{ for } 1 \leq i \leq 8\}$.
- M is a set of 5 membranes with labels from the set $\{1, \ldots, 5\}$.

- $(t_0)_1 =$ 

is the axiom tile present in the membrane m_1.

- $\Sigma = \{\phi\}$.

- $Syn = \{(1,2),(2,3),(3,4),(4,5),(4,1)\}$.

- $T_1 = \{R_{(1,2)}\}$, $T_2 = \{R_{(2,3)}\}$, $T_3 = \{R_{(3,4)}\}$, $T_4 = \{R_{(4,5)}, R_{(4,1)}\}$, $T_5 = \{\phi\}$.

- $R_{(1,2)} = \{[_1(t_i,x)]_1 \to [_2(t_i,x)]_2,$
$[_1(t_i,x)]_1 \to_\mu [_2(t_j,y)]_2$ where $\mu = \{a_i \to b_i, 1 \le i \le 8\},$
$[_1(t_i,x)]_1 \to_\mu [_2(t_j,z)]_2$ where $\mu = \{a_i \to c_i, 1 \le i \le 8\},$
$[_1(t_i,x)]_1 \to_\mu [_2(t_j,l)]_2$ where $\mu = \{a_i \to d_i, 1 \le i \le 8\}\}.$
$R_{(2,3)} = \{[_2(t_i,x)]_2 \to [_3(t_i,x)]_3,$
$[_2(t_i,y)]_2 \to [_3(t_j,y)]_3,$
$[_2(t_i,z)]_2 \to_{(90,x,y)} [_3(t_j,z)]_3,$
$[_2(t_i,l)]_2 \to_{(270,x,y)} [_3(t_j,l)]_3\},$
$R_{(3,4)} = \{(b_4,d_3),(b_5,d_2),(b_6,a_3),(b_7,a_2),(b_8,c_3),(b_1,c_2),(d_1,a_4),(a_1,c_4),$
$[_3t_i]_3 \to [_4t_i]_4\}$
$R_{(4,1)} = \{1(a_7 1 a_8)1 \to 1(a_7)1, 1(a_5 1 a_6)1 \to 1(a_6)1, 1(b_2 1 c_1)1 \to 1(a_2)1,$
$1(b_3 1 d_4)1 \to 1(a_3)1, 1(c_5 1 c_6)1 \to 1(a_8)1, 1(c_7 1 c_8)1 \to 1(a_1)1,$
$1(d_5 1 d_6)1 \to 1(a_4)1, 1(d_7 1 d_8)1 \to 1(a_5)1, [_4(t_i,\phi)]_4 \to [_1(t_i,x)]_1\}$
$R_{(4,5)} = \{1(a_7 1 a_8)1 \to 1(a_7)1, 1(a_5 1 a_6)1 \to 1(a_6)1, 1(b_2 1 c_1)1 \to 1(a_2)1,$
$1(b_3 1 d_4)1 \to 1(a_3)1, 1(c_5 1 c_6)1 \to 1(a_8)1, 1(c_7 1 c_8)1 \to 1(a_1)1,$
$1(d_5 1 d_6)1 \to 1(a_4)1, 1(d_7 1 d_8)1 \to 1(a_5)1, [_4t_i]_4 \to [_5t_i]_5\}$

- m_5 is the output membrane.

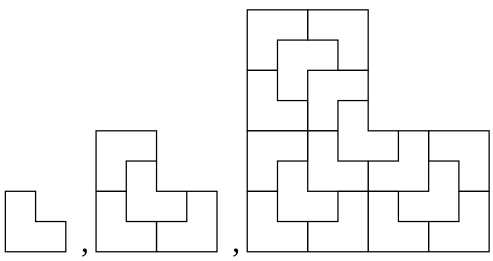


Fig. 4. Chair tiling.

3. Complexity and Efficiency Results

The computational complexity of tissue-like Parametric Tile Pasting P System has been analyzed in terms of the tile types and the geometric operations with the aim of answering the question, which is the minimum number of tile types and geometric operation required for the tiling of the plane or the generation of a pattern. The number of tile types required for the generation of a tiling pattern significantly reduces or increases with the usage or non usage of the geometric operation rotation in the generation of tiling pattern.

Theorem 1. The number of tile types required by a t-PTPPS with rotation is less than the t-PTPPS without rotation.

Proof. Consider a set of primary tiles types $P = \{t_0, \ldots, t_n\}$. For each primary tile $t_i \in P$ let $S = \{t_i^1, t_i^2, \ldots, t_i^{k_j} / i, j \in (0, 1, 2, \ldots, n)\}$ be the set of secondary tiles derived from the primary tiles by rotating the tile t_i. Let $\Sigma = P \cup S$ be the finite set of tile types required for the generation of tiling pattern T_i using a t-PTPPS without geometric operation rotation by the derivation $t_0 \to T_1 \to T_2 \to \cdots \to T_i$. The pattern T_i can be derived by a t-PTPPS with the geometric operation rotation using the tile set $\Sigma' = P$. Suppose that for the derivation of the intermediate patterns ${}_x[T_{n-1}]_x \to {}_y[T_n]_y \to {}_z[T_m]_z$ for $n < i$ and $m \leq i$ we require the secondary tile set $\{t_i^1, t_i^2, \ldots, t_i^{k_j}\}$, $0 \leq i, j \leq n$. This required secondary set of tiles can be generated by applying the geometric operation rotation on the primary tile set. Thus the operation rotation reduces the number of tile types required for the generation of tiling pattern. □

The efficiency of t-PTPPS with various combination of geometric operation is discussed in the following.

Theorem 2. $PL(t\text{-}PTPPS, \phi) \subseteq PL(t\text{-}PTPPS, \alpha)$ where α is the geometric operation rotation on the tiles and tiling patterns.

Theorem 3. $PL(t\text{-}PTPPS, \alpha) \subseteq PL(t\text{-}PTPPS, \alpha, \beta)$ where α is the geometric operation rotation and β is the path rewriting operation.

Theorem 4. $PL(t\text{-}PTPPS, \alpha) \subset PL(t\text{-}PTPPS, \alpha, \beta)$ where α is the geometric operation rotation on the tiles and β is the scaling and edge relabeling operation.

Proof. The self similar tiling pattern constructed in Example 2 cannot be generated by t-PTPPS with rotation alone, as the pattern require the operation of scaling and edge relabeling operation for the fractal boundary of the pattern. □

4. Conclusion

The generation of non-periodic pattern has been studied using t-PTPPS with geometric operations and pasting rules. The family of languages of t-PTPPS with the operation rotation and scaling and edge relabeling operation was found to contain the family of languages of t-PTPPS with rotation alone. Also the t-PTPPS with the operation rotation requires less number of tile types than the t-PTPPS without the operation rotation. The comparison of generation t-PTPPS with Self assembly P systems and other models found in the P systems literature is for our future work.

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