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Evolutionary Dynamic for Inter-Group Cooperation

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Abstract. The evolution of cooperation has been intensively studied in social sciences, biology, and economics mainly using the Prisoner's Dilemma game. Previous studies concentrate on the dynamics of cooperation in a single group. We study the emergence and evolution of cooperation between groups. A new approach of modeling the Prisoner's Dilemma based interactions is presented. A hypergraph representation allowing a detailed study of interaction between groups is considered. Simulations indicate conditions for emergence of inter-group cooperation.

1. Introduction

An important question in Evolutionary Game Theory [4] is the emergence of cooperation. Darwinian evolutionary theory favor defectors, selfish individuals. Despite this, cooperative individuals can benefit from cooperation.

One of the most studied problem in non-cooperative and evolutionary Game Theory is the Prisoner's Dilemma (PD) [1, 5]. This game points the difference between common interest (cooperation), and selfish interest (defection). Two players can choose their strategies: cooperation or defection. If both cooperate they win more than if they both defect, but if one cooperates and the other defects the cooperator wins the worst possible payoff.

The payoffs of the PD game can be summarized in Table 1. Each player has two strategies: cooperate (C) or defect (D). A general description of the payoff functions is described in Table 1. The paradox of the game is obvious with the values $T > R > P > S$ (and $T + S < 2R$).

Table 1. The general payoff functions of the two players in Prisoner's Dilemma

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	(R, R)	(S, T)
	Defect	(T, S)	(P, P)

The public goods game [8] is a generalization of the PD game. In the public goods game the player can choose how much to contribute to a common pool. If everyone gives the maximum contribution, the group has the maximum payoff, but if someone deviates his own payoff will be higher.

Some models that study the emergence of cooperation rely on the Prisoner's Dilemma and on the public goods game. Players are grouped in a spatial framework. In a spatial game the structures usually used to describe the interactions are: lattices, small world networks, scale-free graphs, evolving networks, and random graphs [19].

Interaction between players, in a multi-player system, can be described by a graph. In a spatial game a player does not interact with all other players. Two graphs can be used to describe the interaction between players [16], one that gives the connection between players while the second one describes the learning strategy. To simplify the problem the learning and connection between players can be done by the same graph.

In case of lattices the interactions are defined by the edges of the points whose distance does not exceed a given value. Square lattices are frequently used. For instance every individual plays the Prisoner's Dilemma game with its neighbors, at the end of each round the sites will be occupied by the player with the winning strategy [15] (the player with the highest total payoff). In this manner the winner's strategy spreads.

Within evolutionary graph theory, vertices represent players and the edges of the graph indicate the interactions [11, 17, 19]. The dynamic of the game depends on a specific update rule.

Random graphs may also be used to describe the interaction between players [3, 21]. The idea is to use a framework that allows a more natural interaction between players. The influence of noise can be studied using this structure. For a lattice with fix connectivity the phase diagram (evolution of the game) depends on the lattice and the temptation of players, but is independent of the initial conditions.

Using Prisoner's Dilemma and a spatial structure Nowak and May have studied how cooperation may appear in a homogeneous group [14] (the group is formed by the same type of players). After each round the winning strategy is adopted by the losing players. Tit-for-tat [18] represents an alternative method for changing the players strategy. According to Vainstein and Arenzon [20] cooperation can be promoted through disorder in the spatial structure. Strategy D has a large influence in small-world networks (defectors are enhanced in small-world networks) [20].

Usually in various heterogeneous groups (different player types), the contribution to the public good is lower, compared to homogeneous groups [6, 7, 12]. High levels of cooperation can be achieved in well integrated environments even in well mixed groups. Whereas in a segregated society cooperation does not appear [6, 7]. Increasing group diversity induces cooperation [7].

A mechanism based on self-interest will promote cooperation if the individuals benefits from cooperation. Cooperation can be achieved through a system of punishment. Players have the ability to punish opponents who deviate (do not cooperate) in an integrated environment [12].

Hardin [9] considers that each group can be viewed as a unitary player and intra-group conflict can be modeled with a two-person game. One approach to enhance cooperation between groups is the modification of the payoff matrix [10]. In this case selfish individuals need to consider the common goal [13].

Emergence of cooperation has been studied only for interactions in a group [19]. Our aim is to study the interactions between groups. In order to analyse the emergence of cooperation between heterogeneous groups we concentrate on interactions between groups, ignoring intra-group interactions. A hypergraph interaction model is proposed to describe interactions between players and groups of players. When playing an n-person Prisoner's Dilemma game, an interaction model based on a hypergraph is natural. The interaction between different groups can be described in this way.

The proposed hypergraph model ensures a higher flexibility in the sense of the interactions between players (the players can interact in each round with other random opponents).

2. Concept of hypergraph

For the model representation a hypergraph [2] structure is used. The concept of hypergraph generalizes the notion of a graph: in a hypergraph an edge can connect any number of vertices.

Definition 1. Let $X = \{x_1, x_2, .., x_n\}$ be a finite set, and $E = (E_i, i \in I)$ be a family of subsets of X, satisfying the conditions:

- $E_i \neq \phi, i \in I$;
- $\cup_{i \in I} E_i = X$.

The system $H = (X, E)$ is *a hypergraph*, where X is the set of vertices, and E is the set of hyperedges [2].

3. Proposed model for inter-group cooperation

Interacting groups are represented by a (unique) hypergraph. Vertices of the hypergraph represent the interacting players, a hyperedge represents a group of players. Each player has two possible actions: cooperate or defect. A group is described by a hyperedge.

The PD game is played for a predefined number of rounds. The player type, or strategy (C or D) is initialized randomly. We want to explore the emergence of cooperation between groups, for this reason we investigate only the case when the game is played between hyperedges/groups and not between players from the same hyperedge/group.

We consider several rules for choosing opponents for each player. These rules are the following:

- *Rule* R_1. Each player from a hyperedge plays the Prisoner's Dilemma game with a random number of opponents chosen randomly from the other hyperedges. The opponents are chosen from a single randomly chosen hyperedge at the beginning of each round.
- *Rule* R_2. A predefined number of opponents are randomly chosen for each player from a single other randomly group.
- *Rule* R_3. Each player from one group plays with all players from a single random chosen group.
- *Rule* R_4. Each player has one random opponent.

In a round each player plays PD against his opponents (selected according to a specific rule). The rule does not change in a game. After each round an update step is performed. Players keep their strategy if they have the highest total payoff with respect to their opponents. A losing player adopts the strategy of the most successful wining opponent (the player with the highest total payoff).

The frequency of cooperation in each round is used as a measure of cooperation. Frequency of cooperation (F) in a hyperedge is defined as:

$$F = \frac{\text{number of cooperators}}{\text{number of players}}.$$

4. Numerical experiments

We consider that interacting groups have no common players. This corresponds to disjoint hyperedges.

The payoff settings are $R = 2, P = 1, S = 0, T = b$, b variable (if $b \leq 2$ there is no "dilemma"). Parameter p_D sets the percentage of the defectors initially in the game.

One option is to group players who have the same strategy (C or D) in the same hyperedge, simulating coalitions. Numerical experiments indicate that very soon, one dominant group emerges. The nature of this dominant group, cooperators or defectors, depends on the parameter p_D of the game.

Rules R_1, R_2, R_3, and R_4 are investigated in several experiments.

4.1. Experiment 1: Emerging behaviour using rule R_1

We investigate the case when each player from a hyperedge plays the PD game with a random number of opponents chosen randomly from a unique different hyperedge.

We consider an experiment with three hyperedges, $b = 2.5$, and for the percentage of defectors $p_D = 0.01$. The evolution of frequency of cooperation for the three hyperedges are depicted in Figs. 2, 3 and 4.

Figures 2, 3 and 4 describe two different behaviour. Each Figure describes:

- a situation where the cooperation is established from the first round and remains so;
- a situation where groups adopt defect strategy after fourteen rounds.

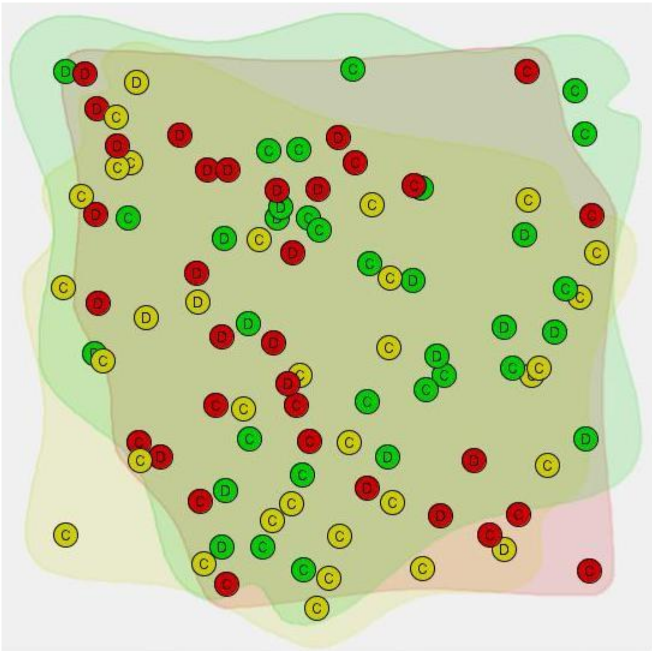

Fig. 1. The hypergraph representation in the fourth round, $p_D = 0.01$, $b = 2.5$. Nodes in hyperedge are identified by different shades.

This can happen due to the random nature of the rule. Let us consider the PD game with $R = 2, S = 0, T = 3, P = 1$. If player p_1 is a cooperator and has five opponents (four cooperators and one defector, for example) his total payoff is *8*. For the opponent p_2, the defector, if it has only two opponents (all cooperators, for example) his total payoff is *6*. In the update process p_2 adopts p_1 strategy. This randomness also explains the transitions from the first *30* rounds from Fig. 5. Figure 5 depicts the result for the case of ten heterogeneous groups, each group having ten players. For a hypergraph having the same number of nodes and two, six, ten hyperedges we obtain similar results.

When the percentage of defectors is greater than 0.01 all players will adopt the defect strategy. It seems that $p_D = 0.01$ is a critical point (describing a phase transition).

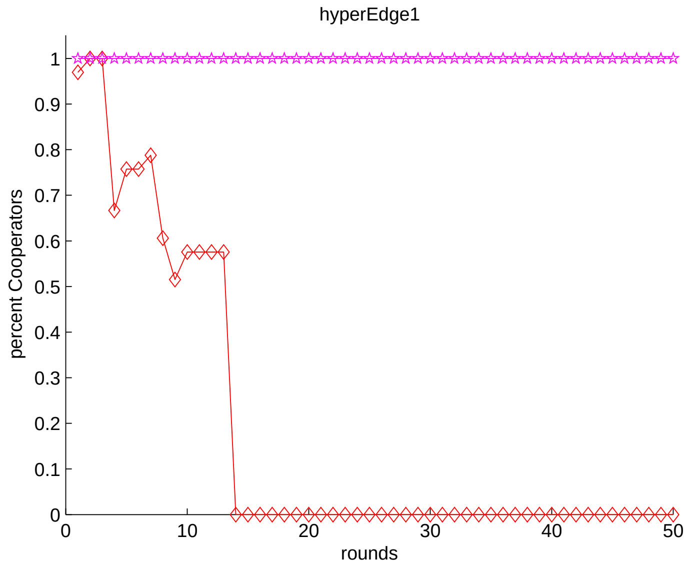

Fig. 2. Evolution of frequency of cooperation in the first hyperedge in two different simulations. Players play according to the rule R_1, $p_D = 0.01$, $b = 2.5$.

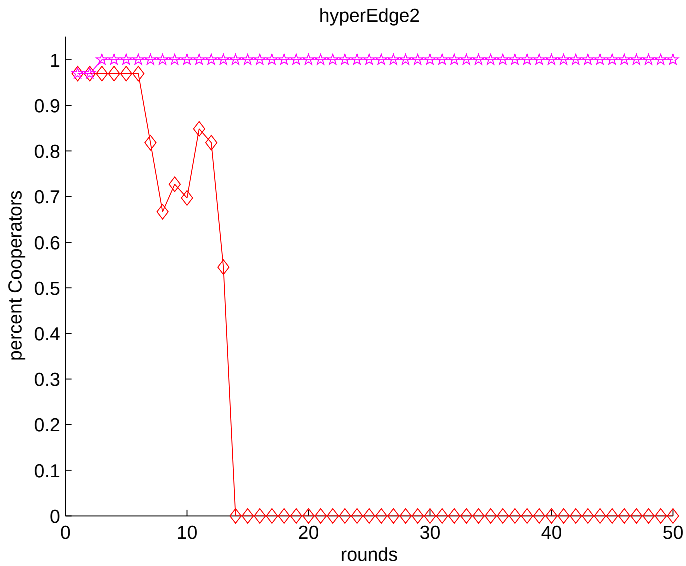

Fig. 3. Evolution of frequency of cooperation in the second hyperedge in two different simulations. Players play according to the rule R_1, $p_D = 0.01$, $b = 2.5$.

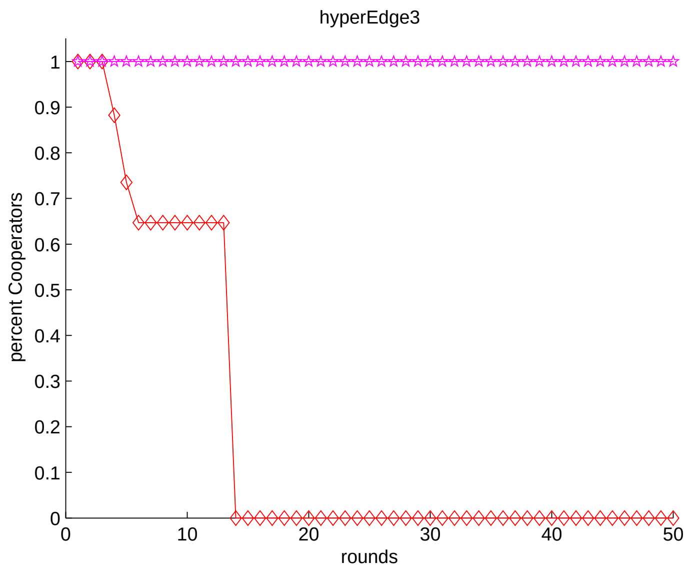

Fig. 4. Frequency of cooperation in the third hyperedge in two different simulations. Players play according to the rule R_1. $p_D = 0.01$, $b = 2.5$.

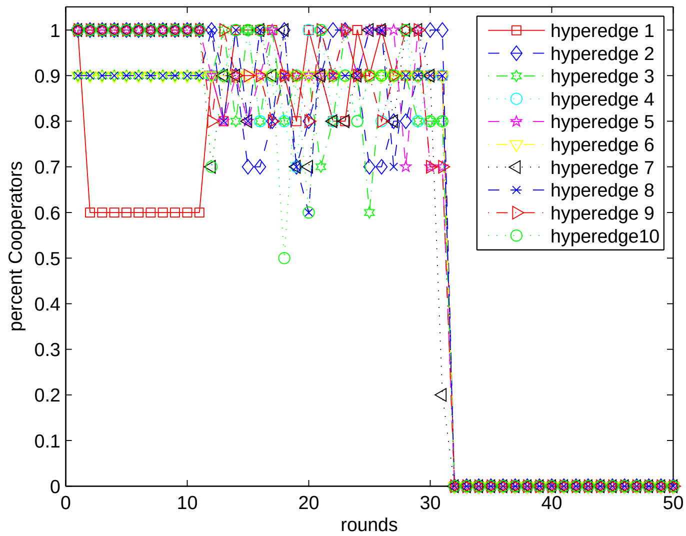

Fig. 5. Evolution of frequency of cooperation for a game played between ten groups, each group has ten players. Players play according to the rule R_1. $p_D = 0.01$, $b = 2.5$. Even if the number of defectors is greater than previous cases (one defector in each group) the groups cooperate for more rounds.

4.2. Experiment 2: Emerging behaviour using rule R_2

In this experiment a fixed number of random opponents are chosen for each player from a unique random group.

Figures 6, 7, 8 and 9 present the influence of parameter p_D and the number of opponents for R_2. By increasing the number of opponents for each player the groups converge to strategy D in fever rounds as depicted in Figures 6 and 7. If we increase the number of defectors in the initial population, as depicted in Figures 8 and 9, strategy defect establishes between the groups after *3-5* rounds. If the proportion of defectors is high ($p_D \geq 0.1$) defection appears between groups.

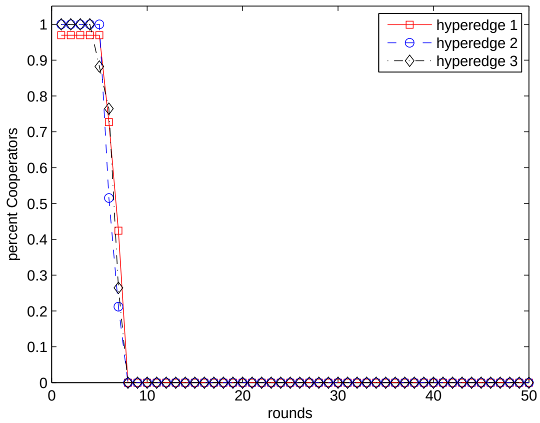

Fig. 6. Evolution of frequency of cooperation for all *3* hyperedges when each player has *4* opponents. Players play according to the rule R_2, $b = 2.5$, $p_D = 0.01$. By increasing the number of opponents for each player the groups converge to strategy D in fewer rounds.

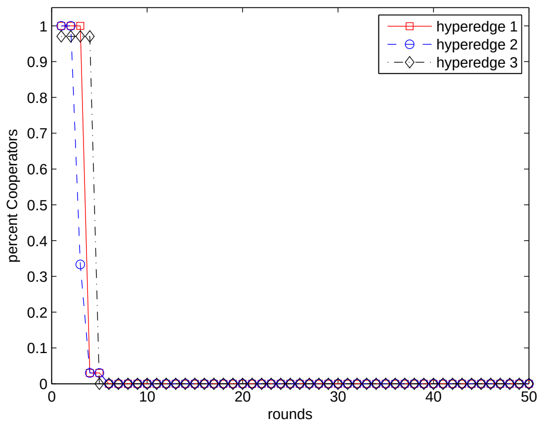

Fig. 7. Evolution of frequency of cooperation for all *3* hyperedges when each player has *29* opponents. Players play according to the rule R_2, $b = 2.5$, $p_D = 0.01$. By increasing the number of opponents for each player the groups converge to strategy D in fewer rounds.

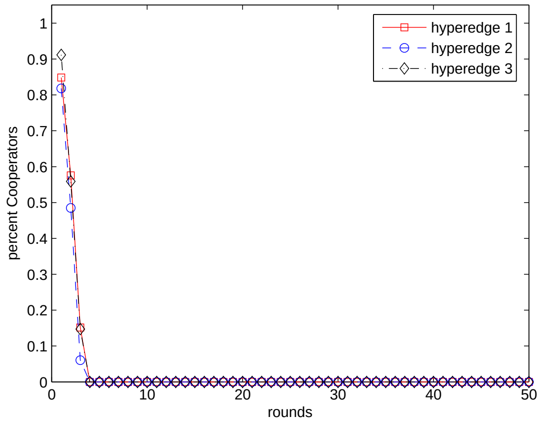

Fig. 8. Evolution of frequency of cooperation for all *3* hyperedges when each player has *4* opponents. Players play according to the rule R_2, $b = 2.5$, $p_D = 0.1$, increasing the number of defectors in the initial population the groups will cooperate for fewer rounds. By increasing the number of opponents for each player the groups converge to strategy D in fewer rounds.

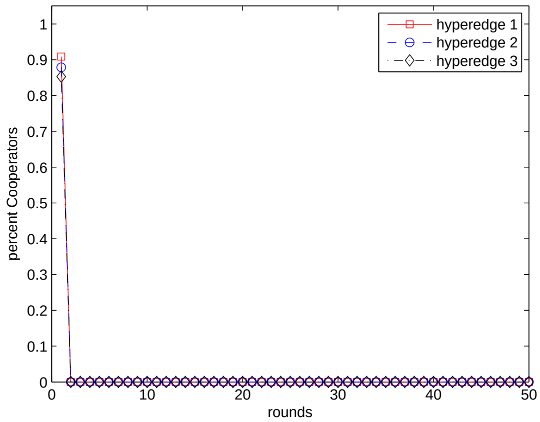

Fig. 9. Evolution of frequency of cooperation for all *3* hyperedges when each player has *29* opponents. Players play according to the rule R_2, $b = 2.5$, $p_D = 0.1$, increasing the number of defectors in the initial population the groups will cooperate for fewer rounds. By increasing the number of opponents for each player the groups converge to strategy D in fewer rounds.

4.3. Experiment 3: Emerging behaviour using rule R_3

In this experiment players from one group play with all members from a group selected at random.

The results are similar to those obtained in R_1 and R_2. Indifferent from the number of hyperedges ($m = 2, 3, 6, 10$) and for different values of b, $b = (2.5, 2.9, 3.3, 3.5, 3.9)$. The cooperation between groups remains stable only for the case when the percentage of defectors is small ($p_D \leq 0.01$).

Figure 10 presents the results obtained for $p_D = 0.01$. When there is a large number of players the strategy defect is the winning one and quickly (in one or two rounds) spreads in the population. Therefore the parameter p_D has little or no influence.

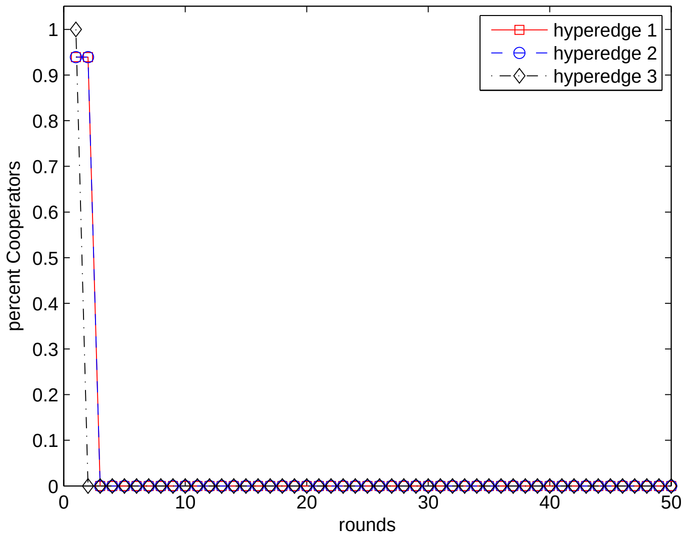

Fig. 10. Evolution of frequency of cooperation for all three hyperedges. Players play according to the rule R_3. $p_D = 0.01$, $b = 2.5$. Because each player plays with all members of a group selected at random, even if the number of defectors in the initial population is small, non-cooperation between groups will establish in a small number of rounds.

4.4. Experiment 4: Emerging behaviour using rule R_4

This experiment investigates a standard situation, each player has one opponent randomly selected from a random hyperedge.

Due to the random nature of the rule, groups will cooperate for more rounds $(30 - 36)$ as shown in Fig. 11.

4.5. Discussions

Evolution of inter-group cooperation is studied using several rules for selecting opponents. Rule R_4 promotes cooperation for *36* rounds because of the low number of opponents in each game. For rules R_1, R_2, and R_3 the groups will cooperate for approximately *10* rounds.

Figures 12 and 13 depict the importance of the number of defectors in the first round. If there is a small number of players with strategy D then the frequency of cooperators is high, when increasing p_D cooperation between groups decreases rapidly.

We study these interactions between a different number of groups ($2-10$ groups). By increasing the number of interacting groups the inter-group cooperation happens for *30* rounds.

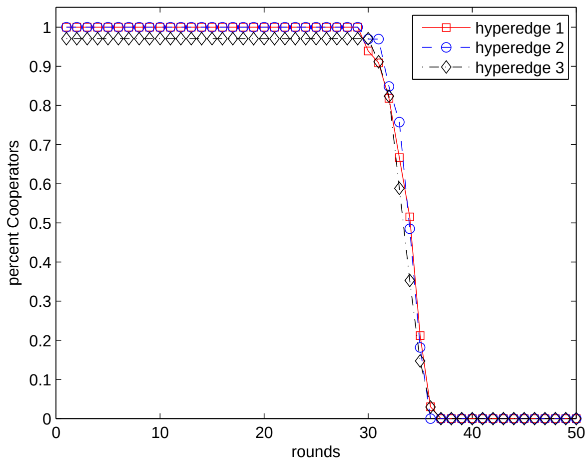

Fig. 11. Evolution of frequency of cooperation for all three hyperedges. Players play according to the rule R_4. $p_D = 0.01$, $b = 2.5$. Due to the random nature of the model, each player has one random opponent from a random hyperedge, groups will cooperate for more rounds. In this case defectors will be involved in fever PD games thus strategy D will spread slower (in *30* rounds).

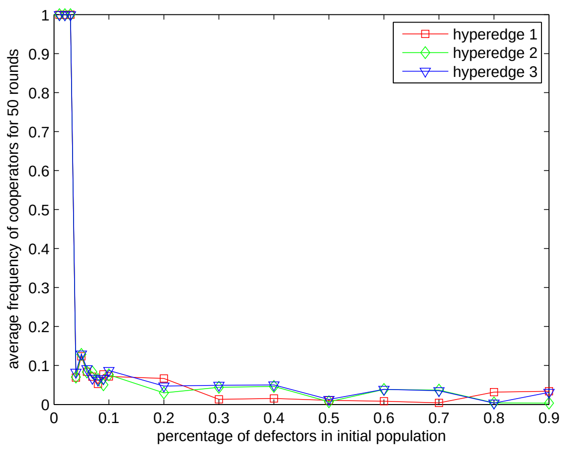

Fig. 12. Importance of parameter p_D for R_1. The number of players that adopt strategy D in the initial population has an important role in the evolution of cooperation between groups.

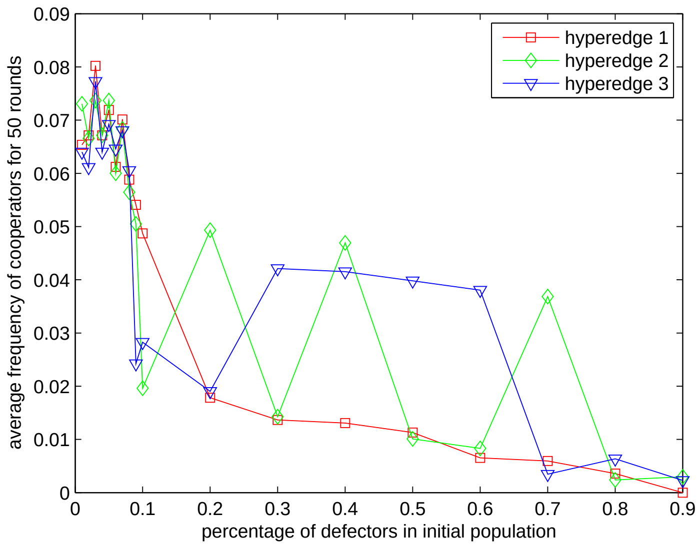

Fig. 13. Importance of parameter p_D for R_2, each player has 6 opponents. The number of players who adopt strategy D in the initial population has an important role in the evolution of cooperation between groups.

5. Conclusions

The interaction inside groups is extensively studied in the literature. Inter-group cooperation seldom happens, and when it does it is much harder to maintain. Our aim is to study the interactions between different groups represented by hyperedges of a hypergraph. We consider the interactions based on four simple rules for choosing opponents in a PD game. Numerical experiments indicate that cooperation between groups arises only in some cases (for a small number of defectors in the initial groups).

The hypergraph model can be useful in describing real-life interactions, and social networks. The hypergraph representation is a natural one for describing intergroup interactions. A hypergraph model allows groups to overlap (as a player may belong to several hyperedges). In a more elaborated model each player can have a random number of opponents from several groups.

Numerical experiments indicate that when the initial number of defectors is small (one or two defectors) cooperation between groups is not affected. Cooperation is independent of the number of groups and the parameter b. When the number of defectors is greater than two, noncooperation between groups can be observed.

The explanation of these results is the dominance of D strategy. If a defector plays PD with some cooperators from other groups the cooperators will adopt the defectors strategy. As this is true for all rounds, the number of defectors increases exponentially. After a few rounds the population is composed only of defectors.

As future work we will explore several update rules, new measures of cooperation between groups and we will investigate some properties of the hypergraphs in order to enhance cooperation.

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