ROMANIAN JOURNAL OF INFORMATION SCIENCE AND TECHNOLOGY
Volume **19**, Numbers 1-2, 2016, 175–187

On the 1/f Noise and Energy Partition in Solid

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Abstract. Fluctuations with 1/f spectrum stand for the fundamental source of decoherence in the nanostructures used to process information in a quantum computer. Consequently, intrinsic 1/f noise of these nanodevices is acting as a roadblock in the field of quantum information and computing. This is another, very stringent reason why finding where this noise comes from became of great practical importance. However, understanding the microscopic mechanism of this ubiquitous phenomenon in solid remained one of the most difficult endeavors since its discovery more than 90 year ago. In this work, five different 1/f noise-related issues are questioned and analyzed from a phenomenological perspective. The aim is to find possible, so-far hidden bridges between them, which might be helpful in grasping the microscopic origin of 1/f noise in solid.

For instance, in deducing his celebrated fluctuation-dissipation theorem, Nyquist assumed the validity of the energy echipartition in an ideal transmission line. We show that for a real system, it is necessary to clarify how the thermal energy is transferred between the eigenmodes. This can happen only if the modes are coupled by microscopic nonlinearities, whose presence confers a dynamic character to the equilibrium state. Therefore, nonlinearity-induced energy exchange between the modes is a condition necessary to have equilibrium fluctuations at the resistor terminals. Thereafter, the empirical procedure Hooge used to get the 1/f noise formula is analyzed. Two unexpected ideas were found behind it. The first one is that 1/f noise cannot exist in the absence of thermal noise, which would suggest an equilibrium microscopic mechanism for 1/f noise. The second one relates to the elimination of the $k_B T$ factor from the thermal noise spectrum. Since $k_B T$ factor is the echipartition fingerprint in thermal noise, its elimination appears as equivalent to the assumption that violation of equipartition is necessary to have 1/f noise. Next, it is shown that deviations of the frequency exponent from 1 must mirror those of the voltage exponent from 2, which means non-validity of the Ohm law.

Therefore, any frequency exponent different from 1 should be considered as the possible macroscopic signature of a microscopic nonlinearity. Examples of deviations from linearity of the 1/f noise for semiconductors, metals and carbonic materials are given.

The experiment done by Voss and Clarke to demonstrate that 1/f noise exists in thermal noise is very briefly described. Its explanation relies on the presence of microscopic nonlinearities. It resulted that the microscopic nonlinearity is a *sine qua non* condition for the existence of both thermal and 1/f noise. The Fermi-Pasta-Ulam paradigmatic experiment is briefly presented in the end. This experiment demonstrating that equipartition law is violated can be of significance to unveil the mechanism of 1/f noise in solid.

1. Introduction

Since the dawn of the information technology era, "communication in the presence of noise", to paraphrase the title of Shannon's milestone paper [1], has been of great scientific and practical interest because noise, as the main enemy of information, was the fundamental factor degrading the capacity of a transmission channel. More recently, one of the modern fields wherein electronic noise plays a fundamental role is the quantum information and computing [2]. The physical process responsible for the quantum computation is the quantum coherence. Obtaining and maintaining the quantum coherence for a long time is a *sine qua non* condition for the quantum information processing. The quantum entanglement with the surrounding environment is one of the most important sources of decoherence. Therefore, methods to decouple the systems processing quantum information from the local electromagnetic sources environmental have been developed [3]. In this way, the intrinsic, material-inherent, noise of the nanodevices and related nanoarchitectures processing the information becomes the fundamental factor of decoherence [3]. In this respect, Paladino and coworkers stated that "there is clear evidence that 1/f noise is detrimental to the required maintenance of quantum coherent dynamics and represents the main source of decoherence" and further added: "Material inherent fluctuations with 1/f spectrum represent the main limiting factor to quantum coherent behavior of the present generation of nanodevices."[3]. It results that fluctuations with 1/f spectrum are able to completely ruin the advantage of quantum algorithms. Therefore, the interest in finding the cause of the 1/f noise in solid is stringent, for this phenomenon acts as a roadblock in the evolution of the quantum computing.

The 1/f noise conundrum is as old as the quantum mechanics itself. Soon after the invention of the vacuum tubes [4], it was found that the performance of the electronic circuits is affected by noise mechanisms intrinsic to the tubes and materials used in their realization. Already before the first World War, there were problems with a kind of electronic noise generated by the electrons flow in the tube, latter known as shot noise. Also, at Bell Laboratories, the technicians knew that the noise of the amplifiers with vacuum tubes was dependent on the value of the resistance at the input [20]. Since the incumbent technical aspects were of strategic importance during the war, the publication in 1918 of the famous Schottky's paper on the shot noise theory [5] came as a surprise. This effect was of great practical and technological interest at that time because shot noise was considered to act as a fundamental limit for the amplification capability of the vacuum tubes.

In search for the shot noise predicted by Schottky [5], Johnson [6] not only confirmed the existence of this effect but also observed that under a given frequency (f), the intensity of the noise spectrum does not remain flat (white) but starts increasing, while the frequency decreases. In a first attempt to model the new effect, Schottky called it "flicker noise" [7], by analogy with the flickering stars or candles. In the first decade after its discovery in vacuum tubes, 1/f noise, as it was called later, has been observed in a metal (platinum) [8] and graphitic materials [9]. Thereafter, in the next 80 years, it was found in innumerable solid-state physical systems [10, 11, 12, 13, 14, 15] and solid-state electronic devices [16]. No low-frequency limit has ever been observed, although measurements have been done till very low frequencies ($< 10^{-6}$Hz) [17, 18]. This is why, by analogy with the Rayleigh-Jeans law (ultraviolet catastrophe) of blackbody radiation, the phenomenon is also known as the infrared catastrophe. The microscopic origin of 1/f noise in solid is still investigated for more than 90 years. So far, innumerable theoretical approaches were not successful in explaining where this ubiquitous noise comes from.

In this work, we heuristically analyze apparently dissimilar aspects related to the 1/f noise in solid, with the purpose to reveal possible hidden but unifying factors involved in the microscopic mechanism of the 1/f noise in solid.

2. Nyquist thermal noise formula: microscopic nonlinearities in equilibrium

The existence of thermal fluctuations (noise) of electricity in conductors has been predicted by Einstein in his celebrated theory of the Brownian motion [19]. These equilibrium fluctuations are the result of charge redistribution in a resistor due to thermal agitation of the molecular environmental. The phenomenon has been experimentally observed in 1927 by Johnson [20] and theoretically explained by Nyquist [21], who found that the spectral density (S_{th}) of the thermal noise electromotrice force is given by the formula:

$$S_{th} = 4k_BTR, \tag{1}$$

where k_B is the Boltzmann constant, R is the value of the resistance and T is the absolute temperature. The Nyquist formula pertains to a larger class of relations known under the generic name of fluctuation-dissipation theorem [22], [23], which states that "every source of fluctuations is associated with a mechanism of dissipation" [24]. In Kubo's words, "this theorem states a general relationship between the response of a given system to an external disturbance" [23], in this case the resistance R, "and the internal fluctuation of the system in the absence of the disturbance." [23], which is the spectral density (S_{th}) in the relation (1).

In deducing the relation (1), Nyquist [23] relied on two fundamental assumptions: 1- the validity of the second law of thermodynamics and 2 – the validity of the energy equipartition among the natural vibration frequencies of an ideal transmission line connecting two resistors of the same value (R), at the same temperature, therefore in thermodynamic equilibrium. According to the equipartition law, "to each degree of freedom there corresponds an energy equal to kT on the average, *on the basis of the equipartititon law,* where k is the Boltzmann constant." [23]. As the relation (1) shows, at a constant temperature, the response of the system does not depend of the measurement frequency. This is the consequence of the validity of the energy equipartition. Since the experiments agree with the thermal noise formula, it can be taken as a confirmation of the equipartition hypothesis. However, as Nyquist mentioned, at high frequency, namely in the quantum regime, the thermal noise spectrum starts depend on the frequency and this is mainly due to the nonvalidity of the equipartition law. In this case, the energy redistribution among the various proper vibration modes should be described by a Planck spectrum. Therefore, in some circumstances, one can state that deviation of the thermal noise spectrum from a flat one could be the signature of the nonvalidity of the equipartition hypothesis. In this context, the following question is raised: is it possible to have energy echipartition violated at low frequency, too?

A fundamental open question in Nyquist theory is what the mechanism contributing to the repartition of the energy between the modes would be, so that to reach "an energy equal to kT *on the average*" [23]. In fact, how the energy is distributed among the degrees of freedom so as to reach equilibrium is the longstanding, fundamental problem of the ergodic theory in statistical physics and the subject of innumerable papers and controversies [25], [26]. In terms of the ideal Nyquist model, a perfect distribution of the thermal energy among all degrees of freedom, *with no energy transfer between them after that*, would generate a frozen state of the system. Apparently, if no energy is flowing between the modes, there would be no potential fluctuations at the resistor terminals. If the equipartition between all degrees of freedom was reached, then why is the system fluctuating? Does is not the state of equilibrium, therefore of maximum entropy, a dead state of the system? Abbott *et al.* observed that thermal equilibrium cannot be a rest state because "it would be a violation of the Uncertainty Principle" [27] and added that noise "...is a manifestation of a dissipative system maintaining thermal equilibrium...." [27].

In our opinion, the system is continuously trying to reach the equipartition state which is spontaneously violated by the energy transfer between its or part of its degrees of freedom. Therefore, the fluctuations at the terminals of a resistor in equilibrium could be viewed as the macroscopic manifestation of the perpetual effort the system is made at the microscopic scale to reach the equipartition or ergodicity state. If this observation is correct, it would support the idea that the thermal equilibrium has a dynamic character. This is in agreement with what Pippard stated about Johnson noise: "…fluctuations are not to be regarded as spontaneous departures from equilibrium configuration of a system, but are manifestations of the dynamic character of thermal equilibrium itself." [28]

If so, it results that the thermal equilibrium is not a dead state of the system and fluctuations at the resistor terminals cannot occur without energy transfer between different vibration modes. But the echipartition cannot be reached in the absence of the mode coupling, therefore without microscopic nonlinearity. In this respect, it might be of interest to mention that the high frequency mode-coupling has also been invoked in a model of 1/f noise [29]. One also notes that Mandelbrot and Voss [30] consider nonlinearity as a *sine qua non* condition for 1/f noise generation in the fractal model of the 1/f noise. Altogether, our interpretation converges toward the idea that thermal noise cannot exist in the absence of the nonlinear interactions between the degree of freedom of the system under observation.

3. Cancelling the effect of equipartition in thermal noise: Hooge's formula

Collecting many existing experimental data from literature, mainly for semiconductors, Hooge [31] found that 1/f noise spectral density (S_V) follows an inverse dependence on the total number of carriers (N) in the investigated sample. Next, using dimensional arguments, he tried to model this observation by a kind of renormalization of the thermal noise ($4k_BTR$) in a resistor R:

$$S_V = 4k_BTR(1 + \gamma v_d^2/Df), \tag{2}$$

where the first term, $4k_BTR$, is the thermal noise in the sample and the second one, $4k_BTR\gamma v_d^2/Df$, is the spectral density of the 1/f noise. In this relation, k_B is the Boltzmann constant, T – absolute temperature, R – sample resistance, v_d – carrier drift velocity, D – diffusion coefficient, f is the frequency and γ is a parameter. With the well-known relations: $R = L/e\mu\text{nS}$, $v_d = \mu E$, $E = V/L$ and $D = k_B\text{T}\mu/e$ for resistance, drift velocity, electric field and diffusion coefficient, respectively, where e is the elementary charge, μ-electron mobility, n-carrier concentration in resistor, L -resistor length, S - cross section of the resistor, E - electric field and V is the voltage across the resistor terminals, one finally gets:

$$S_V/V^2 = \alpha/Nf, \tag{3}$$

where α=4γ and N=nLS is the total carrier number in resistor. This formula was deduced by Hooge strictly from dimensional considerations, with the purpose to describe the experimental dependence of the 1/f noise on the total carrier number. The *ad hoc* introduction of the drift velocity points to the fact that Eq. 3 describes 1/f noise as a non-equilibrium effect. It cannot explain the existence of the 1/f noise in equilibrium [47], therefore in the absence of an applied electric field (v_d=0). Moreover, the frequency is also artificially introduced, namely as a factor to account for the time in the ratio between a dissipative factor (drift velocity) and the diffusion coefficient, which, usually, describes the equilibrium fluctuations.

The Hooge approach suggests that under non-equilibrium 1/f noise coexists with the thermal noise at any frequency, but it is visible only below a threshold frequency, while above it the 1/f noise is lost in the thermal noise. Moreover, although controversial, the Hooge's empirical procedure supports the idea that 1/f noise cannot exist in the absence of thermal noise. As can be seen from the formula (2), the factor k_BT in the thermal noise is replaced by introduction of the diffusion coefficient D=k_BTμ/e. Bell considered that "the elimination of diffusion is questionable" [24]. On the other hand, in Hooge's approach, the 1/f noise formula (3) cannot be obtained without replacement of the k_BT factor. As we have seen, the occurrence of this factor in the thermal noise spectrum is dictated by the validity of the equipartition law. Therefore, two mandatory factors are needed to get 1/f noise formula. The existence of thermal noise is the first one. In this context, 1/f noise appears as intrinsic to the thermal noise. The second one is to cancel the effect of equipartition. Consequently, the elimination of the k_BT factor from thermal noise to get 1/f noise formula would be equivalent with the assumption of non-validity of the equipartition theorem. If correct, this observation suggests that the fluctuations with 1/f spectrum could be a manifestation of the violation of the equipartition law in solid. The idea that 1/f noise mechanism in solid could be related to the non-validity of the echipartition theorem has been discussed, more than two decades ago, with Prof. Musha [32]. The fundamental theoretical aspects of this fascinating hypothesis have been developed, both classically and quantum mechanically, in a series of very interesting papers by Musha and coworkers [33]-[35].

4. Signature of microscopic nonlinearity in 1/f noise: violation of the Ohm law

Relation (3) shows that 1/f noise intensity features a quadratic dependence on voltage, $S_V \sim V^2$. This is what the linear response theory predicts: a quadratic dependence of the noise intensity on the excitation factor (voltage, current). With Callen and Welton's definition: "the system may be said to be linear if the power dissipation is quadratic in the magnitude of the perturbation" [22]. In fact, this result expresses the validity of the Ohm law in the field of electronic noise. Since the Ohm law is valid, $S_V \sim R^2I^2$, where I is the current through the sample, it turns out that 1/f noise is due to resistance fluctuations. In the case of the Ohm law, "the current does not generate the resistance but is necessary to measure it" [14]. Similarly, "in the case of 1/f noise the current does not generate the resistance fluctuations but is necessary to have an observable noise voltage" [14].

As formula (3) shows, for a strictly linear system, the frequency exponent must be exactly 1. The trouble is that the frequency exponent is rarely 1. This is the so-called 1/f-like noise, $f^{1\pm\eta}$, with the exponent, $(1\pm\eta)$, between approx. 1.3 and 0.7. If Hooge formula (3) is complete and applicable to 1/f-like noise, dimensional arguments ask for the voltage exponent to deviate from 2:

$$S_V/V^{2\pm\beta} = \alpha/Nf^{1\pm\eta}. \tag{4}$$

Therefore, Callen and Welton's definition presented above is violated and the system behavior is no more linear. According to the relation (4), any deviation of the frequency exponent from 1 must be accompanied by a similar deviation of the voltage exponent from 2. Ideally, $\beta = \eta$ always, but a one-by-one correlation between the two exponents is very difficult to prove experimentally and, in fact, it has never been done. Frequency exponents slightly different from 1 are usually observed experimentally, pure 1/f noise being barely found. When the voltage dependence of the noise intensity is investigated, only a few points are considered enough to draw a conclusion on the validity of the Ohm law in that material or device. If any, slight deviations of the exponent from 2 are usually overlooked, the main purpose being to show that the system follows the Ohm's law. In fact, and especially when the frequency exponent differs from 1, the goal would be to show that this law is violated, not vice-versa. There are, however, enough data showing that the noise mechanism is nonlinear, some of them being presented in what follows.

For instance, in carbon microphone and other granular resistance, Christensen and Pearson [9] reported that for different resistors, the voltage exponent varies between 1.75 and 1.97, with an average of 1.85. This is a consistent sublinear dependence of the 1/f noise on voltage in these materials. In the case of an 800 Å thick silver film on sapphire, Duta and Horn [14] found that the voltage noise $S_V \sim V^{2.26\pm0.13}$, which stands for a clear deviation from the Ohm law in a metal. These authors consider that this is "the most disturbing from our results" [14]. Undoubtedly, this is a macroscopic signature in 1/f noise of a nonlinear microscopic mechanism. In evaporated films of InSb and InAs, Epstein [36] reported that 1/f noise power featured an $I^{1.6}$ current dependence, while for an InSb sample, obtained by another method, the noise intensity varied as $\propto I^{2.1}$. It indicates that the noise mechanism in these materials slightly deviates from the linear response theory (Ohm's law).

In an individual carbon nanotube, Roche *et al.* [37] reported strong deviation from I^2 law. Similar results for individual carbon nanotubes have been observed by Roumiantsev *et al.* [38]. In multiwalled carbon nanotube bundles, a strong peak in the noise intensity has been reported as a function of voltage at very low temperatures (4.2K) [39]. Consistent deviations from the Ohm law have been found in the normalized 1/f noise intensity (S_I/I^2) by Back *et al.* [40] around the energy corresponding to a Kohn anomaly in metallic carbon nanotubes, at room temperature. These results point to the fact that deviations from the Ohm law manifest merely at some given voltages and that the presence of nonlinearity can be easier revealed when the noise spectral density is normalized to V^2 or I^2. In fact, such deviations from the Ohm law in the dependence of the 1/f noise intensity on voltage have been reported long before in different materials, such as GaAs tunneling Schottky diode [41], metallic point contacts [42], two-dimensional electron gas [43] and carbon soot resistors [44]. One notes, however, that the first two systems are definitely not linear, while in the last two the Ohm law is fully vindicated.

One can thus state that there are enough data to consider that the nonlinearity is involved in the 1/f noise mechanism. Apparently, it is looked for in the dependence of the noise intensity on voltage. However, if it is to give credence to the 1/f-like noise formula, more direct evidence would come from the frequency exponent, for, in the hypothesis that the formula (4) is complete, it should mirror the voltage exponent. Therefore, within the experimental errors, any deviation of the frequency exponent from 1 can be the macroscopic signature of a microscopic nonlinear mechanism, but this is not a common practice.

5. 1/f noise in thermal noise: Voss and Clarke experiment

Voss and Clarke [45] have had the idea to spectrally analyze the fluctuations of the variance of thermal noise voltage in some semiconductor films and metals. To this goal, very small InSb resistors, with about 10^6 atoms, have been manufactured. Similar niobium films have been investigated. These authors found that in equilibrium, therefore in the absence of a current flowing through the sample, the fluctuations in the variance of thermal noise feature 1/f noise spectra. In addition, the films have been measured under different biasing conditions: dc current, ac and pulsed technique (to reduce the power dissipated in the sample). To exclude the influence of the external sources, the InSb resistor was replaced by a metal resistor of the same value, but with no 1/f noise. A similar methodology has been used to evaluate the noise spectrum of the fluctuations in a niobium film.

For both semiconducting and metallic film, the spectra measured under nonequilibrium conditions were almost identical with the spectrum obtained in equilibrium. It indicates that neither current flowing through the resistor, nor the power dissipation is the cause of the 1/f noise. This fundamental experiment definitely demonstrated that 1/f noise exists in or coexists with the thermal noise. Therefore, exactly as the thermal noise, it is an equilibrium phenomenon. Since 1/f noise is intrinsic to the thermal noise, a current flowing through the resistor does not generate but only reveals it. Variation of the noise intensity with the voltage or current at the power of two is the signature of the resistance fluctuations. Therefore, the resistance fluctuation is the common source of both thermal noise and 1/f noise. Similar results have been reported for carbon samples [46].

The theoretical explanation of this experimental result is very challenging. Tremblay and Nelkin showed that "the essential physics of the problem is contained in mode-coupling terms..." [47], [48] and then added that it leads "to a very small fluctuation *renormalization of* the average resistance and average *Johnson-noise power*" [47], [48]. It is to be reminded that a kind of thermal (Johnson) noise renormalization was used by Hooge to arrive at the 1/f noise formula. Although used merely as a *Deus ex machina*, it is of interest to note that Hooge's empirical procedure is in the spirit of Tremblay and Nelkins' "renormalization of the... average Johnson noise power" [47], [48]. Moreover, "mode-coupling terms" are equivalent with the existence of the nonlinearities in equilibrium. According to Nelkin and Tremblay, the nonlinearity in the equation of motion is mandatory in order to have 1/f noise in equilibrium. But nonlinearity is the factor we have invoked to explain the echipartition hypothesis used by Nyquist in deduction of the thermal noise spectrum. It would result that neither thermal noise nor equilibrium 1/f noise exists in the absence of microscopic nonlinearity. Therefore, the microscopic nonlinearity is a *sine qua non* condition for the generation of both thermal and 1/f noise.

6. Violation of the equipartition theorem: Fermi-Pasla-Ulam experiment

Under nonequilibrium conditions, Teitler and Osborne considered that when energy is introduced into a system by a dc source, "the noise would presumably be caused by the necessity for energy introduced on a macroscopic scale to be dissipated over a range down to a microscopic scale" [49]. Next, "the presence of nonlinear processes" [49] was considered responsible for the energy dissipation which "occurs over a range of wave numbers including those as high as the inverse of typical microscopic mean free path." [49].

This statement is strikingly similar to the hypothesis Fermi has started from, almost two decades earlier, in his famous Fermi-Pasta-Ulam (FPU) paradigmatic experiment [50]. The goal of the numerical FPU experiment was to validate the echipartition in a unidimensional, fixed-end chain of N echi-mass particles, when the interaction potential contains nonlinear terms. Such a test vehicle is the equivalent of a one dimensional crystal [51]. The first mode of the system was initially excited and the evolution of the system was numerically calculated for a time interval of 16 periods of the fundamental mode [52]. The expectation was that due to the nonlinear terms in the potential of interaction, energy induced in the first mode will be redistributed among all modes of the particles string, so that equipartition, therefore the statistical equilibrium, will be reached. The surprise was that the energy was shared only among the first five modes, the other "modes 6 through 32 were forever left lying in the "noise" gasping for energy" [52]. FPU computational experiment showed that even in the presence of nonlinear interaction, the approach to equipartition is lacking. Moreover, this experiment revealed that there is energy exchange only between the low frequency modes of the system, while the high frequency modes are excluded. This observation could be relevant for the mechanism of 1/f noise. In this respect, Fukamachi reported 1/f noise in a FPU lattice at frequencies lower than 10^{-2} Hz [53]. He found "that 1/f fluctuations do not come from the large number of degrees of freedom" [53]. In this sense, he showed that for small nonlinearity, 1/f noise exists even in very small system (N=8). However, the Fukamachi's results were not completely vindicated by Kawamura *et al.* [54], who found merely a Lorentzian spectrum of the phonon number fluctuations in a FPU lattice.

The FPU experiment is fascinating for it shows that nonlinearity is not enough to have the energy equally distributed among the all modes. However, as established by Izrailev and Chirikov [55], the equipartition is rapidly approached if the initial excitation energy is larger than a certain threshold. It would result that below this threshold, the role of nonlinearity is paradoxical for it is of help in the energy transfer but not enough to distribute it equally among all modes. This observation is of special interest when the system is in or close to equilibrium. Finally, the FPU experiment shows that the answer to the previous question whether equipartition can be violated at low frequencies is positive. It can be useful to understand the mechanism of 1/f noise in solid.

7. Conclusions

The fundamental hypotheses the Nyquist fluctuation-dissipation theorem relies on have been reappraised. It has been found that the introduction of the microscopic nonlinearity is necessary to have energy equally distributed among the proper vibration modes of the transmission line he used to deduce the thermal noise spectrum. The idea that nonlinearity is the cause of the equilibrium fluctuations has been introduced. The empirical approach used by Hooge to get the 1/f noise formula was analyzed. The idea that 1/f noise cannot exist in the absence of thermal noise has been enounced. Also, the elimination of the $k_B T$ factor from the thermal noise has been evaluated as a possible signature of the equipartition violation. Within this frame, it has been suggested that 1/f noise is intrinsic to thermal noise, whenever the equipartition theorem is not valid. The formula for 1/f-like noise has been discussed and the significance of the frequency exponent different from 1 was considered as the signature of the microscopic nonlinearity. The Voss and Clarke experiment was presented, with the main purpose to evidence that the occurrence of the 1/f noise in thermal noise can be explained only if the existence of the microscopic nonlinearity in equilibrium is taken into consideration. The Fermi-Pasta-Ulam experiment has been described to show that even in the presence of the nonlinearity the energy equipartition between the low-frequency modes is not reached, which is of relevance for the mechanism of 1/f noise in solid. Our analysis revealed that nonlinearity and non-validity of the echipartition theorem are the main unifying factors between different 1/f noise-related issues investigated in this work.

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